

Walking on the Ladder: 125 GeV Technidilaton, or Conformal Higgs

-Dedicated to the late Professor Yoichiro Nambu-

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The walking technicolor based on the ladder Schwinger-Dyson gap equation is studied, with the scale-invariant coupling being an idealization of the Caswell-Banks-Zaks infrared fixed point in the “anti-Veneziano limit”, such that $N_C \rightarrow \infty$ with $N_C \cdot \alpha(\mu^2) = \text{fixed}$ and $N_F/N_C = \text{fixed}$ ($\gg 1$), of the $SU(N_C)$ gauge theory with massless N_F flavors *near criticality*. We show that the 125 GeV Higgs can be naturally identified with the technidilaton (TD) predicted in the walking technicolor, a pseudo Nambu-Goldstone (NG) boson of the spontaneous symmetry breaking of the approximate scale symmetry. Ladder calculations yield the TD mass M_ϕ from the trace anomaly as $M_\phi^2 F_\phi^2 = -4\langle\theta_\mu^\mu\rangle = -\frac{\beta(\alpha(\mu^2))}{\alpha(\mu^2)} \langle G_{\lambda\nu}^2(\mu^2) \rangle \simeq N_C N_F \frac{16}{\pi^4} m_F^4$, independently of the renormalization point μ , where m_F is the dynamical mass of the technifermion, and $F_\phi = \mathcal{O}(\sqrt{N_F N_C} m_F)$ the TD decay constant. It reads $M_\phi^2 \simeq (\frac{v_{\text{EW}}}{2} \cdot \frac{5v_{\text{EW}}}{F_\phi})^2 \cdot [\frac{8}{N_F} \frac{4}{N_C}]$, ($v_{\text{EW}} = 246$ GeV), which implies $F_\phi \simeq 5 v_{\text{EW}}$ for $M_\phi \simeq 125$ GeV $\simeq \frac{1}{2} v_{\text{EW}}$ in the one-family model ($N_C = 4, N_F = 8$), in good agreement with the current LHC Higgs data. The result reflects a generic scaling $M_\phi^2/v_{\text{EW}}^2 \sim M_\phi^2/F_\phi^2 \sim m_F^2/F_\phi^2 \sim 1/(N_F N_C) \rightarrow 0$ as a vanishing trace anomaly, namely the TD has a mass vanishing in the anti-Veneziano limit, similarly to η' meson as a pseudo-NG boson of the ordinary QCD with vanishing $U(1)_A$ anomaly in the Veneziano limit ($N_F/N_C \ll 1$).

I. INTRODUCTION

The Higgs boson with mass nearly 125 GeV has been found at LHC. Still there remains a mystery about the electroweak symmetry breaking, or the dynamical origin of the Higgs, which would be understood by physics beyond the Standard Model (SM). An attractive idea for the origin of mass beyond the SM is the *dynamical symmetry breaking traced back to Nambu* [1], the birthplace of all the variants of the concept of the spontaneous symmetry breaking (SSB). Together with the Nambu’s dynamical symmetry breaking producing composite Nambu-Goldstone (NG) bosons, we should mention the composite approach by Shoichi Sakata, who proposed the Sakata model [2], a composite model for the hadrons, which paved a way to the quark model and eventually to the Standard Model. We are inspired by his never-ending enthusiasm seeking the deeper level of matter.

In contrast to the SM Higgs boson which has a mass given ad hoc without explanation, the origin of mass M in the Nambu’s dynamical symmetry breaking resides in the *criticality* with the *nonzero critical coupling* $g_{\text{cr}} \neq 0$, such that the value M in the Nambu-Jona-Lasinio model (NJL) is generated from nothing as $M \sim \Lambda (1/g_{\text{cr}} - 1/g)^{1/2}$ for *strong coupling* $g > g_{\text{cr}}$, where g and Λ are the dimensionless coupling and an intrinsic scale carried by the four-fermion coupling, respectively, as $G \sim g/\Lambda^2$. We all know now that the Nambu’s great idea is essentially realized in the reality, the QCD, where the strong gauge coupling in the infrared scale Λ_{QCD} gives rise to the hadron mass on that scale. In the case of Higgs, the top quark condensate model (Top-mode standard model) [3–5] is a straightforward application of the NJL dynamics, with only the top coupling set to be above the criticality and others below it [3, 5] ^{#1}: The origin of the intrinsic scale Λ could be the quantum mechanical origin as the trace anomaly like Λ_{QCD} in the classically scale-invariant theory e.g, gauge theory, or the explicit one such as the given four-fermion coupling in the NJL with Λ to be regarded as the Landau pole or the compositeness scale, or the intrinsic scale of certain underlying gauge theory at deeper level. Note that the existence of the scale Λ does not necessarily implies the existence of the mass M : The weak coupling $g < g_{\text{cr}}$ does not produce the mass M , while the strong coupling does create it, picking

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^{#1} Note that the NJL dynamics with $g_{\text{cr}} \neq 0$ is *in sharp contrast to the weakly-coupled BCS theory which has $g_{\text{cr}} = 0$* due to the “dimensional reduction” by the presence of the Fermi surface. The NJL criticality $g^{\text{top}} > g_{\text{cr}} > g^{\text{others}}$ is an essence of the top quark condensate model of Ref.[3, 5] to ensure that only the top quark gets condensed to produce only three NG bosons to be absorbed into the weak bosons.

up the intrinsic scale Λ a la dimensional transmutation, generically in the form $M \sim \Lambda f(g(\Lambda)) = \mu f(g(\mu))$, with $f(g(\mu)) \rightarrow 0$ as $g(\mu) \rightarrow g_{\text{cr}}$.

One of the candidates for such a dynamical symmetry breaking theory beyond the SM is the walking technicolor (WTC) [6–8], having a *large anomalous dimension* $\gamma_m = 1$ ^{#2} to solve the Flavor-Changing Neutral Currents (FCNC) problem ^{#3} of the original technicolor (TC) [16, 17] and a *technidilaton* (TD), a pseudo NG boson of the approximate scale symmetry, as a composite Higgs (“Conformal Higgs” [18]) ^{#4}. It has recently been shown that the TD properties are consistent with the current data of LHC for the 125 GeV Higgs and hence TD can be identified with the 125 GeV Higgs at LHC [20–23].

The above results [6, 7] were originally obtained based on the ladder Schwinger-Dyson (SD) gap equation for the fermion propagator, with the nonrunning gauge coupling constant $\alpha(\mu^2) = \alpha$ for $0 < \mu^2 < \Lambda^2$ as an input coupling: The theory is scale-invariant (infrared conformality) in the infrared region below the cutoff Λ to be identified with the intrinsic scale of the theory, Λ_{TC} , like Λ_{QCD} of ordinary QCD, which is quantum mechanically induced by the regularization as the trace anomaly. When the coupling exceeds the critical coupling $\alpha > \alpha_{\text{cr}} \neq 0$, the chiral and scale symmetries simultaneously get SSB due to the generation of the technifermion dynamical mass m_F in such a way that m_F is much smaller than the intrinsic scale $m_F \sim \Lambda_{\text{TC}} f(\alpha) \ll \Lambda_{\text{TC}}$ by the Miransky scaling [24] (similar to Berezinsky-Kosterlitz-Thouless (BKT) scaling), with $f(\alpha) \rightarrow 0$ ($\alpha \rightarrow \alpha_{\text{cr}}$) in the essential-singularity form, thus retaining the approximate scale invariance $\alpha(\mu^2) \approx \alpha$ for the wide infrared region $m_F^2 < \mu^2 < \Lambda_{\text{TC}}^2$. The generation of the tiny m_F in units of the intrinsic scale Λ breaks the scale symmetry explicitly as well as spontaneously, so that the TD as a pseudo NG boson was expected to acquire a tiny mass to be estimated by the anomalous Ward-Takahashi (WT) identity for the approximate scale symmetry via Partially Conserved Dilatation Current (PCDC) relation [7], in the same manner as the pion mass estimate by the Partially Conserved Axial Current (PCAC).

Since then, the WTC has confronted other challenges, namely, the S, T, U parameters [25] from the electroweak precision measurements ^{#5}, the large top quark mass 173 GeV ^{#6}, and finally the most serious and urgent problem from the discovery of the Higgs at 125 GeV. This created a widely spread folklore against TC including WTC: e.g., “More intuitively, the measured mass of the Higgs tells us that it is weakly coupled. Strong coupling solutions like Technicolor tend to lead to a strongly coupled Higgs” [26].

This is totally a misconception based on the linear sigma model, whose $\lambda|\phi|^4$ coupling for the would-be QCD σ meson with mass $M_\sigma^2 = 2\lambda v^2 \simeq (6v)^2 \simeq (500 \text{ MeV})^2$ would be obviously strong $\lambda \sim (6v)^2/(2v^2) \simeq 18 \gg 1$, in sharp contrast to the 125 GeV Higgs with $\lambda \simeq (125 \text{ GeV})^2/[2(246 \text{ GeV})^2] \simeq 1/8 \ll 1$. Actually, the linear sigma model is not the right effective theory of QCD, rather the nonlinear sigma model corresponding to λ or $M_\sigma^2 \rightarrow \infty$ is the correct one, the Chiral Perturbation Theory (ChPT). The ChPT is not scale-invariant, which is in accord with the QCD having no scale invariance. However, the WTC does have an approximate scale invariance and hence its effective field theory must be approximately scale-invariant. The light composite Higgs, the TD as the pseudo NG boson of the approximate scale symmetry, does make the nonlinear sigma model (approximately) scale-invariant, in a way fully consistent with the strongly coupled underlying theory, the WTC (“scale-invariant ChPT”, sChPT for short) [21, 27]. It will be shown in Eq.(106) that the self-interactions of the TD are even weaker than the SM Higgs!

Note that all the bound states (techni-hadrons) are in principle strongly coupled to each other *within the WTC sector* just as hadrons in QCD are, whereas the couplings of 125 GeV Higgs so far observed at LHC are *not those among the techni-hadrons* but only the couplings of a *special technihadron*, TD, to the SM sector particles, which must be weak, through either the (weak) $SU(2) \times U(1)$ gauge couplings or the (weak) effective Yukawa couplings (loop-suppressed and extended TC (ETC)-scale suppressed via ETC-like couplings), see Eq.(57), all related to outside of the strongly coupled WTC sector. Moreover, it was shown that the TD couplings themselves characterized by $1/F_\phi \ll 1/v_{\text{EW}}$, are *even weaker* than those of the SM Higgs [22, 23]. See Eqs.(106) and (108).

Another widely spread misconception comes from the ChPT, the opposite to the linear sigma model view. It says that there is no light composite scalar meson with mass much lighter than the scale of the naive dimensional analysis (NDA), $4\pi F_\pi$, which is based on the estimated breakdown scale of the conventional ChPT valid in the ordinary QCD.

^{#2} It was further shown [9] that the NJL model coupled to the walking gauge theories (“gauged NJL model”) has an even larger anomalous dimension $1 < \gamma_m < 2$, along the critical line [10, 11] with strong four-fermion coupling. It was further shown [12] that such a theory is renormalizable without Landau pole, i.e., nontriviality theory having a finite nontrivial (non-Gaussian) ultraviolet fixed point, in contrast the pure NJL model which is a trivial theory. See later discussions.

^{#3} Solving FCNC problem by a large anomalous dimension was proposed earlier [13], based on a pure assumption of the existence of a gauge theory having the nontrivial UV fixed point at large coupling, where a large anomalous dimension $\gamma_m > 1$ was postulated. See also [14, 15].

^{#4} Similar works on the FCNC solution [19] were done without notion of the anomalous dimension, the scale invariance, and the technidilaton.

^{#5} There are several solutions to the S parameter problem. See the discussions in the last section.

^{#6} Possible resolutions are discussed in the last section.

The crucial assumption of NDA is that no light spectrum other than the pions exist below $4\pi F_\pi$, which however is already in contradiction with the reality even in the conventional QCD: $M_{f_0} \simeq 500 \text{ MeV}$ and $M_\rho \simeq 770 \text{ MeV}$, well below the NDA $4\pi F_\pi \simeq 1.2 \text{ GeV}$. Actually, the statement should be reversed: If there exists a light spectrum lighter than $4\pi F_\pi$, then the conventional ChPT should be modified so as to include the light spectrum in such a way that the effective theory must respect the symmetry of the underlying theory. In the case at hand, it is the sChPT [21, 27].

There have been much progress of the WTC particularly on the light TD, not just in the ladder SD equation, but also in a variety of approaches such as the ladder Bethe-Salpeter (BS) equation combined with the ladder SD equation [28, 29], the effective theory based on the sChPT [21, 27], with possible extension including vector mesons via HLS in a scale-invariant manner [30], holographic method [23, 31, 32], and eventually, the first-principle calculation of the flavor-singlet scalar meson in the large N_F QCD on the lattice [33–36]. In particular, it is remarkable that such a light flavor-singlet scalar meson as a candidate for the TD was observed in the lattice $N_F = 8$ QCD [33], the theory shown to have signatures of the lattice walking theory including the anomalous dimension $\gamma_m \simeq 1$ [37–39]. Note that $N_F = 8$ (four weak-doublets) corresponds to the “one-family model” [17, 40] which is the most straightforward model building of the ETC [41] as a standard way to give mass to the quarks and leptons. The one-family model of the WTC with $N_C = 4$ is in fact best fit to the 125 GeV Higgs data [20–23], and is shown to be most natural for the ETC model building [42].

Among such many approaches, the ladder SD equation is still a powerful and relatively handy tool to analyze the TD as a composite Higgs, in spite of the fact that it is not a systematic approximation in the sense that high order corrections are not controllable (see below, however). In fact it turned out to be more than a mnemonic of the physics guess: It well reproduced numerically as well as the qualitatively the nonperturbative aspects of the ordinary QCD in the hadron physics, with additional ansatz simply replacing the nonrunning coupling by the one-loop running one as the input coupling of the SD equation [43]. Many ladder analyses on the dynamical symmetry breaking with large anomalous dimensions in the strongly coupled gauge theories and gauged NJL model gave many suggestive results in the applications for WTC, top quark condensate model, etc.. [8, 43].

In this paper, in the new light of the 125 GeV Higgs at LHC, we investigate full implications of the ladder SD gap equation for the WTC, in the context of near conformal window of large N_F QCD, $SU(N_C)$ gauge theory with massless N_F flavors [44, 45], in the particular walking limit, “anti-Veneziano limit” (in distinction to the original Veneziano limit with $N_F/N_C \ll 1$):

$$N_C \rightarrow \infty \quad \text{and} \quad \lambda \equiv N_C \cdot \alpha = \text{fixed}, \quad \text{with} \quad r \equiv N_F/N_C = \text{fixed} \gg 1, \quad (1)$$

(See [30, 32] for preliminary discussions). Such a limit realizes the ideal situation for the ladder SD equation, where the input perturbative coupling becomes nonrunning (infrared conformality), $\alpha(\mu^2) \equiv \alpha_*$, thanks to the perturbative infrared (IR) fixed point (Caswell-Banks-Zaks (CBZ) IR fixed point) [46], $\alpha(\mu^2) \simeq \alpha_* = \alpha(\mu^2 = 0)$ already near the anti-Veneziano limit for $0 < \mu^2 < \Lambda_{\text{TC}}^2$. See Fig.1. The present paper is an extension of Ref.[47], where a similar analysis was done without concept of the anti-Veneziano limit.

In the ladder SD equation, the critical coupling α_{cr} is given as $\alpha_{\text{cr}} = \frac{\pi}{3C_2}$, with the quadratic Casimir $C_2 = \frac{N_C^2 - 1}{2N_C}$. For the strong coupling $\alpha = \alpha_* > \alpha_{\text{cr}}$, the technifermion acquires the dynamical mass m_F in an essential-singularity (non-analytic) form a la Miransky-Berezinsky-Kosterlitz-Thouless [24] of the conformal phase transition [45]:

$$m_F \simeq 4\Lambda \cdot \exp\left(-\frac{\pi}{\sqrt{\frac{\alpha}{\alpha_{\text{cr}}} - 1}}\right) \ll \Lambda \quad \left(0 < \frac{\alpha}{\alpha_{\text{cr}}} - 1 \ll 1, \quad \alpha_{\text{cr}} = \frac{\pi}{3C_2} = \frac{\pi}{3} \frac{2N_C}{N_C^2 - 1}\right), \quad (2)$$

which implies a large hierarchy $m_F \ll \Lambda$ near the criticality $\alpha \simeq \alpha_{\text{cr}}$, where the cutoff Λ as a regulator may be regarded as the intrinsic scale $\Lambda = \Lambda_{\text{TC}}$. The would-be CBZ IR fixed point α_* is washed out by the mass, which however is a small mass $m_F \ll \Lambda_{\text{TC}}$, so that there still remains an approximate scale symmetry in a wide infrared region $m_F < \mu < \Lambda_{\text{TC}}$. Note that in the walking/anti-Veneziano limit the ladder approximation becomes more trustable, since the coupling becomes “weak”,

$$\alpha(\mu) \simeq \alpha_* \simeq \alpha_{\text{cr}} \sim 1/N_C \rightarrow 0, \quad (3)$$

so that many non-ladder diagrams without C_2 factor multiplied on α are suppressed as in the usual $1/N_C$ expansion (also is the case in the NJL model where $(g, g_{\text{cr}}) \sim 1/N_C$), in spite of the fact that the 't Hooft coupling λ is really strong and the “effective” critical coupling to trigger the chiral condensate is strong, $C_2\alpha_{\text{cr}} = \frac{\pi}{3} > 1$ such that $\lambda > \lambda_{\text{cr}} = (N_C/C_2)\pi/3 \rightarrow 2\pi/3$.

Eq.(2) dictates that α is no longer constant due to $m_F \neq 0$ but does run depending on Λ/m_F according to the

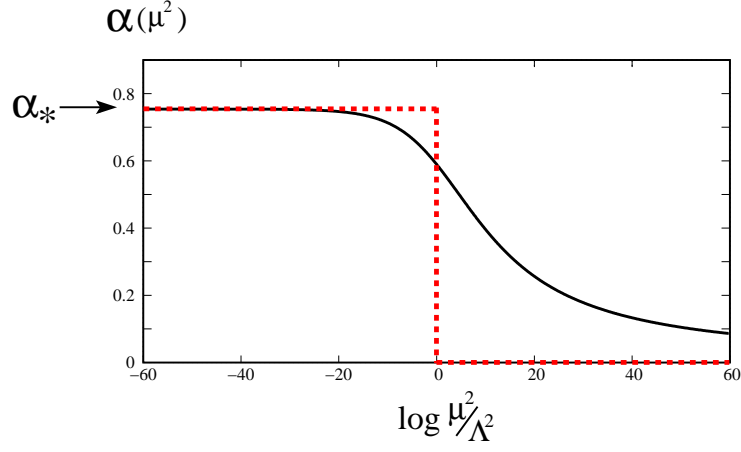


FIG. 1: Two-loop running coupling (solid curve) in the case of $SU(3)$ gauge theory with $N_F = 12$ massless fundamental fermions with intrinsic scale Λ_{TC} , compared with the ladder coupling for $\Lambda = \Lambda_{TC}$. The overall scale of $\alpha(\mu)$ shrinks like $1/N_C$ to zero in the large N_C limit with $N_C\alpha = \text{fixed}$ (i.e., $\Lambda = \Lambda_{TC}$ is fixed) and $r = N_F/N_C = \text{fixed} \gg 1$ (walking/anti-Veneziano limit).

nonperturbative beta function $\beta^{(NP)}(\alpha)$ [48] (See Fig. 1 (a) of Ref. [6]):

$$\beta^{(NP)}(\alpha) = \Lambda \frac{\partial \alpha(\Lambda)}{\partial \Lambda} = -\frac{2\pi^2 \alpha_{cr}}{\ln^3\left(\frac{4\Lambda}{m_F}\right)} = -\frac{2\alpha_{cr}}{\pi} \left(\frac{\alpha}{\alpha_{cr}} - 1\right)^{\frac{3}{2}} \quad (\alpha > \alpha_{cr}), \quad (4)$$

as $\Lambda/m_F \rightarrow \infty$, and hence the coupling as a solution of $\frac{\partial \alpha}{\partial \ln \mu} = \beta^{(NP)}(\alpha)$ runs as renormalization point μ :

$$\alpha(\mu) = \alpha_{cr} \left[1 + \frac{\pi^2}{\ln^2\left(\frac{\mu}{\mu_{IR}}\right)} \right] \quad (\alpha(\mu) > \alpha_{cr}), \quad (5)$$

with $\mu_{IR} = \mathcal{O}(m_F)$, even when the perturbative coupling (input coupling) is nonrunning, $\beta(\alpha)|^{\text{perturbative}} \equiv 0$. This is completely different from the two-loop beta function having the CBZ IR fixed point, which is no longer valid for $\alpha > \alpha_{cr}$, where $\alpha(\mu) \searrow \alpha_{cr}$ ($\mu \nearrow$): α_{cr} is now regarded as the ultraviolet (UV) fixed point, as was emphasized in Ref. [6] in the context of the WTC. Then the would-be *IR fixed point* $\alpha_* \simeq \alpha_{cr}$ is also regarded as the *UV fixed point* of the nonperturbative running (walking) coupling $\alpha(\mu) \approx \alpha_{cr}$ in the wide IR region $m_F < \mu < \Lambda = \Lambda_{TC}$ for the characteristic large hierarchy (“criticality hierarchy”) $m_F \ll \Lambda_{TC}$ [47, 49]. (See also Ref.[50] for a similar observation.)

The scale symmetry is broken also explicitly by m_F which is generated by the SSB of the the same scale symmetry, the typical order parameter being the decay constant of the TD, F_ϕ , defined as $\langle 0|D_\nu|\phi(q)\rangle = -iF_\phi q_\nu$ ^{#7}. Different from the SSB of the internal symmetry like chiral symmetry, there exists no exact point where the scale symmetry is spontaneously broken without explicit breaking. Nevertheless there exists a limit where the explicit breaking is much smaller than the SSB scale of the scale symmetry, that is, the walking/anti-Veneziano limit, Eq.(1).

Note that m_F is an N_F, N_C -independent quantity related to $\Lambda = \Lambda_{TC}$ via Miransky scaling, Eq.(2), with $\alpha/\alpha_{cr} \simeq \alpha_*/\alpha_{cr}$ being only dependent of the ratio $r = N_F/N_C$ in the anti-Veneziano limit. Since the dilatation current is a sum of all the N_F and N_C - technifermion species, $D_\nu(x) \sim N_F N_C$, and the TD state is normalized as $|\phi\rangle \sim 1/\sqrt{N_F N_C}$, we have $F_\phi \sim \sqrt{N_F N_C} m_F$ by definition of F_ϕ , so that the explicit breaking m_F is much smaller than F_ϕ , or the NDA associated with the TD loop $(4\pi F_\phi)^2/N_F$ in the walking/anti-Veneziano limit, in addition to the criticality hierarchy $m_F \ll \Lambda_{TC}$ of direct relevance to the scale symmetry:

$$m_F^2 \ll F_\phi^2, \quad \frac{(4\pi F_\phi)^2}{N_F} \ll \Lambda_{TC}^2. \quad (6)$$

^{#7} F_ϕ is also defined as $\langle 0|\theta_{\mu\nu}|\phi(q)\rangle = F_\phi(q_\mu q_\nu - q^2 g_{\mu\nu})/3$, which yields the identical F_ϕ .

Then the mass of the TD as a pseudo-NG boson M_ϕ can be evaluated, based on the anomalous WT identity for the scale symmetry as the PCDC relation for the trace anomaly [7] :

$$\begin{aligned} M_\phi^2 F_\phi^2 &= -F_\phi \langle 0 | \partial_\mu D^\mu | \phi \rangle^{(NP)} = -4 \langle 0 | \theta_\mu^\mu | 0 \rangle^{(NP)} = -\frac{\beta^{(NP)}(\alpha(\mu))}{\alpha(\mu)} \langle G_{\nu\lambda}^2(\mu) \rangle^{(NP)} \\ &= N_F N_C \left(\frac{16\xi^2}{\pi^4} m_F^4 \right), \quad (\xi \simeq 1.1), \end{aligned} \quad (7)$$

where the last expression was given through the ladder evaluation of the vacuum energy $E = \langle 0 | \theta_\mu^\mu | 0 \rangle^{(NP)} / 4$ [51]. Since $F_\phi^2 \sim N_F N_C m_F^2$ by definition of F_ϕ , Eq.(7) is also in accord with the fact that M_ϕ^2 as well as m_F^2 has no explicit dependence on N_F and N_C . Here all the quantities with $(\cdot)^{(NP)}$ to be defined later contain only the nonperturbative contributions arising from the dynamical mass $m_F \neq 0$ due to the SSB, and hence vanishes as $m_F \rightarrow 0$.

We show that independent calculations of $\frac{\beta^{(NP)}(\alpha(\mu))}{4\alpha(\mu)}$ and $\langle G_{\nu\lambda}^2(\mu) \rangle^{(NP)}$ in the ladder approximation yield the nonperturbative trace anomaly as a product of them, which precisely agrees with the result calculated independently from the vacuum energy of Ref.[51]. The agreement is realized in a highly nontrivial manner, fully consistent with the Renormalization-Group Equation (RGE) point of view: Each of the $\frac{\beta^{(NP)}(\alpha(\mu))}{4\alpha(\mu)}$ and $\langle G_{\nu\lambda}^2(\mu) \rangle^{(NP)}$ does depend on the renormalization point μ : See Eq.(4) for $\frac{\beta^{(NP)}(\alpha(\mu))}{4\alpha(\mu)} \sim 1/\ln^3 \mu$, and explicit calculation reads $\langle G_{\nu\lambda}^2(\mu) \rangle^{(NP)} \sim \ln^3 \mu$ for $m_F < \mu < \Lambda_{TC}$. Such a μ -dependence is completely cancelled each other in the product to arrive at the μ -independent trace anomaly as it should be. Similar cancellation of the μ -dependence also takes place in the ordinary QCD, where $\frac{\beta(\alpha(\mu))}{4\alpha(\mu)} \sim 1/(\ln \mu)$ and $\langle G_{\nu\lambda}^2(\mu) \rangle \sim \ln \mu$, with the logarithm of $\ln \mu$ instead of $\ln^3 \mu$. This result is the RGE view of the previous calculation [47] based on the improved ladder approximation for the large N_F $SU(N_C)$ gauge theories near conformal window.

The key observation of the present paper is that as far as the PCDC relation is satisfied as in the ladder approximation, the TD as a pseudo NG boson has a vanishing mass in the anti-Veneziano limit, quite independently of the numerical details of the ladder calculation (See [30, 32] for preliminary discussions): Noting that $F_\phi^2 = \mathcal{O}(N_F N_C m_F^2)$, Eq.(7) naturally explains

$$M_\phi = \mathcal{O}\left(\frac{4\xi m_F}{\pi^2}\right) = \mathcal{O}\left(\frac{m_F}{2}\right) \ll F_\phi = \mathcal{O}(\sqrt{N_F N_C} m_F), \quad (8)$$

in the walking/anti-Veneziano limit, Eq.(1), where we have $M_\phi^2/F_\phi^2 \sim 1/(N_F N_C) \rightarrow 0$. The TD as the pseudo NG boson has a vanishing mass limit, though not exact massless point, in the anti-Veneziano limit, where the nonperturbative trace anomaly vanishes in units of F_ϕ as a measure of the SSB of the scale symmetry. This is similar to the η' meson in QCD, which is regarded as a pseudo NG boson whose mass, evaluated through the anomalous WT identity with the $U(1)_A$ anomaly, does vanish in the large N_F and N_C limit with N_F/N_C fixed ($\ll 1$) (Veneziano limit): $M_{\eta'}^2/F_\pi^2 \sim N_F/N_C^2 \rightarrow 0$, without the exact massless point.

Note that m_F is related to the weak scale $v_{EW} = 246$ GeV through the Pagels-Stokar formula $F_\pi^2 \simeq (N_C \xi^2 / 2\pi^2) m_F^2$ in the ladder approximation reads (Eq.(B4)): $v_{EW}^2 = (246 \text{ GeV})^2 = N_D F_\pi^2 \simeq \frac{N_F N_C \xi^2}{4\pi^2} m_F^2 \simeq m_F^2 \left[\frac{N_F}{8} \frac{N_C}{4} \right]$, with $N_D (= N_F/2)$ being the number of the electroweak doublets. Then a natural estimate of the TD mass for the one-family model $N_F = 8$ with $N_C = 4$ is that $M_\phi = \mathcal{O}(m_F/2) = \mathcal{O}(v_{EW}/2) = \mathcal{O}(125 \text{ GeV})$, in agreement with the LHC Higgs as the TD. More precisely, Eq.(7) can be rewritten in terms of v_{EW} :

$$M_\phi^2 \simeq \left(\frac{v_{EW}}{2} \right)^2 \cdot \left(\frac{5 v_{EW}}{F_\phi} \right)^2 \cdot \left[\frac{8}{N_F} \frac{4}{N_C} \right]. \quad (9)$$

It was first pointed out in Ref. [20] that this ladder PCDC result accommodates the 125 GeV Higgs with $F_\phi = \mathcal{O}(\text{TeV})$ for the one-family model with $N_F = 8$ and was shown to be the best fit to the current LHC data:

$$F_\phi \simeq 5 v_{EW} \simeq 1.25 \text{ TeV} \quad \text{for} \quad M_\phi = 125 \text{ GeV} \quad (N_F = 8, N_C = 4) \quad (10)$$

[20–23] (See also the later discussions). With the fact that $v_{EW}^2 \propto N_F N_C m_F^2 \sim F_\phi^2$, the result reflects the generic scaling:

$$\frac{M_\phi}{v_{EW}} \sim \frac{M_\phi}{F_\phi} \sim \frac{m_F}{F_\phi} \sim \frac{1}{\sqrt{N_F N_C}} \rightarrow 0, \quad (11)$$

in the anti-Veneziano limit ^{#8}.

On the other hand, all the non-NG boson technihadrons, such as the techni-rho, techni- a_1 , technibaryon, etc., have *no constraints from the PCDC as the explicit breaking of the scale symmetry* but do have *constraints from the SSB of the scale symmetry*, so that they should have masses on the scale of SSB of the scale symmetry, characterized by F_ϕ much larger than $2m_F$ of the naive nonrelativistic quark model picture:

$$M_\rho, M_{a_1}, M_N, \dots = \mathcal{O}(\text{TeV's}) > \mathcal{O}(F_\phi) \gg 2m_F \gg M_\phi. \quad (12)$$

In fact, the IR conformal physics of the WTC should be described by the low-lying composite fields as effective fields, in a way to realize all the symmetry structure of the underlying theory.

Such an effective theory of WTC as a straightforward extension of sChPT [21, 27] is already constructed, i.e, the scale-invariant version [30] of the Hidden Local Symmetry (HLS) model [55, 56], (the “sHLS model”), where the technirho mass terms have the *scale-invariance nonlinearly realized* by the TD field $\chi = e^{\phi/F_\phi}$, with the SSB of the scale invariance characterized by the scale of F_ϕ , while the Higgs (TD) mass term in the TD potential, on the order of $m_F (\ll F_\phi)$, is the only source of the explicit breaking of the scale symmetry related (via PCDC) to the nonperturbative trace anomaly of the underlying theory.

One interesting candidate for such technihadrons may be a resonance behind the diboson excess recently observed at LHC at 2 TeV [57, 58], which can be identified with the walking technirho [59]. A smoking gun of the walking techni-rho is the absence of the decay to the 125 GeV Higgs (TD), which is forbidden by the scale symmetry explicitly broken only by the Higgs (TD) mass term (corresponding to the nonperturbative trace anomaly in the underlying WTC) [60]. Actually, the salient feature of the scale symmetry of the generic effective theory not just the sHLS model, containing the SM gauge bosons and the Higgs plus new vector bosons (any other massive particles as well), is the *absence of the decay of the new vector bosons* such as the technirho (and also other higher resonances) *into the 125 GeV Higgs plus the SM gauge bosons* [60]. If such decays of new particles are not found at LHC Run II, then the 125 GeV Higgs is nothing but the dilaton (TD in the case of the WTC) responsible for the nonlinearly realized scale symmetry, i.e., the SSB of the scale symmetry, no matter what underlying theory may be beyond the SM. This should be tested in the ongoing LHC Run-II.

The paper is organized as follows: In section II we review the solutions of the ladder SD equation in some details and the conformal phase transition a la Miransky-BKT in the context of CBZ IR fixed point of the large N_F QCD in the anti-Veneziano limit. Nonperturbative beta function and the corresponding running of the coupling is discussed. Large anomalous dimension $\gamma_m = 1$ and its phenomenological implications are reviewed. In section III we explicitly show the RG invariance of the nonperturbative trace anomaly in the broken phase of the ladder SD equation, in such a way that three independent calculations of $\frac{\beta^{(NP)}(\alpha)}{4\alpha}$, $\langle G_{\mu\nu}^2 \rangle^{(NP)}$ and $\langle (\theta_\mu^\mu) \rangle^{(NP)}$ yield precisely a correct trace anomaly relation. We further check explicitly that the ladder calculation satisfies the anomalous WT identity in the case of nonzero fermion mass $m_0 \neq 0$. This is to establish the consistency of the ladder calculation with the sChPT proposed in Ref.[27] for determining the mass M_ϕ and the decay constant F_ϕ of the TD on the lattice. In section IV we give a mass and decay constant of the TD through the PCDC relation as an anomalous WT identity for the scale symmetry based on the nonperturbative trace anomaly. We discuss that the TD becomes a parametrical NG boson in the anti-Veneziano limit in accord with the walking regime of large N_F QCD, in a sense similar to the η' meson in the ordinary QCD a la Witten-Veneziano. In section V we show that the light TD is consistent with the current LHC data on the 125 GeV Higgs, as an update of the Ref. [22]. Section VI is for the technihadrons other than TD. Section VII is devoted to summary and discussions. Appendix A is for the basic formulas of the ladder SD equation. In Appendix B we give a ladder result for the Pagels-Stokar formula for F_π^2 . Appendix C is for the details about the contamination of the 125 GeV Higgs production between the gluon fusion production and the vector boson fusion production at the present LHC data.

^{#8} In early days, ladder-like calculations [48, 51–54] both in pure gauge theories and gauged NJL models showed $M_\phi = \mathcal{O}(m_F)$, however without paying attention to the N_F, N_C dependence, particularly to the fact that $\frac{M_\phi}{v_{EW}} \sim \frac{m_F}{v_{EW}} \ll 1$ in the anti-Veneziano limit, which is actually of the most phenomenological relevance with respect to the 125 GeV Higgs such as in the one-family model with $N_F = 8, N_C = 4$.

II. SOLUTION OF THE LADDER SD EQUATION AND CONFORMAL PHASE TRANSITION

A. Ladder coupling as the CBZ IR fixed point in the anti-Veneziano limit

Let us first recapitulate the results in Ref.[6] based on the the ladder SD gap equation for the technifermion mass function $\Sigma(-p^2)$ ($p^2 < 0$) with the nonrunning coupling as an idea limit of the CBZ IR fixed point of large N_F $SU(N_C)$ gauge theories, which can be well described by the improved ladder approximation with the running coupling $g^2(-p^2)$ [61]:

$$S_F^{-1}(p) = S^{-1}(p) + \int \frac{d^4 k}{(2\pi)^4} C_2 g^2((p-k)^2) D_{\mu\nu}(p-k) \gamma^\mu S_F(k) \gamma^\nu, \quad (13)$$

where $iS_F^{-1}(p) = Z^{-1}(-p^2)(\not{p} - \Sigma(-p^2))$ and $iS^{-1}(\not{p} - m_0)$ are the full and bare technifermion propagators, respectively, and $iD_{\mu\nu}(p)$ the bare technigluon propagator in the Landau gauge, with an ansatz $g^2((p-k)^2) \Rightarrow g^2(\max\{-p^2, -k^2\})$. C_2 is the quadratic Casimir of the technifermion of the gauge theory, with $C_2 = (N_C^2 - 1)/(2N_C)$ for the fundamental representation in $SU(N_C)$. After the angular integration, the improved ladder SD equation in Landau gauge for $\Sigma(x \equiv -p^2)$ reads:

$$\Sigma(x) = m_0 + \frac{3C_2}{4\pi} \int dy \left[\frac{\alpha(x)}{x} \theta(x-y) + \frac{\alpha(y)}{y} \theta(y-x) \right] \frac{y\Sigma(y)}{y + \Sigma^2(y)}, \quad (Z^{-1}(x) \equiv 1). \quad (14)$$

The original ladder SD gap equation as the basis for the WTC [6, 7] is a scale-invariant dynamics, having an input coupling as *nonrunning*:

$$\alpha(x) = \frac{g^2(x)}{4\pi} \equiv \alpha, \quad \beta(\alpha) \equiv 0, \quad \text{for } 0 < x < \Lambda^2. \quad (15)$$

The cutoff Λ breaks explicitly the scale symmetry, as does the the intrinsic scale Λ_{TC} analogous to the Λ_{QCD} . Such a scale-invariant coupling is indeed an idealization of the CBZ IR fixed point [46] $\alpha = \alpha_*$, such that $\beta^{(2-loop)}(\alpha_*) = 0$ and $\alpha(\mu^2) \approx \alpha_*$ ($\mu^2 \ll \Lambda_{\text{TC}}^2$) in the large N_F QCD [44, 45], where the two-loop coupling is almost nonrunning particularly in the walking/anti-Veneziano limit Eq.(1), while it is rapidly decreasing in the one-loop dominated asymptotically free UV region $\mu > \Lambda_{\text{TC}}$, as in the ordinary QCD (See Fig.1):

$$\begin{aligned} \mu \frac{\partial}{\partial \mu} \alpha &= \beta^{(2-loop)}(\alpha) = -b_0 \alpha^2 - b_1 \alpha^3, \\ b_0 &= \frac{1}{6\pi} (11N_C - 2N_F), \quad b_1 = \frac{1}{24\pi^2} \left(34N_C^2 - 10N_C N_F - 3 \frac{N_C^2 - 1}{N_C} N_F \right), \\ \alpha_* &= -\frac{b_0}{b_1} \longrightarrow \frac{4\pi}{N_C} \frac{11-2r}{13r-34} \quad \left(N_C \rightarrow \infty, \frac{34}{13} < r \equiv \frac{N_F}{N_C} = \text{const} < \frac{11}{2} \right). \end{aligned} \quad (16)$$

The analytic form of α is given as

$$\alpha(\mu^2) = \frac{\alpha_*}{1 + W(z(\mu))}, \quad z(\mu) \equiv \frac{1}{e} \left(\frac{\mu}{\Lambda_{\text{TC}}} \right)^{b_0 \alpha_*}, \quad \text{with } b_0 \alpha_* \longrightarrow \frac{2}{3} \frac{(11-2r)^2}{34-13r}, \quad (17)$$

where $W(z)$ is the Lambert W function and $\Lambda = \Lambda_{\text{TC}}$ is the intrinsic scale defined as

$$\Lambda_{\text{TC}} = \mu \cdot \exp \left(- \int^{\alpha(\mu^2)} \frac{d\alpha}{\beta^{(2-loop)}(\alpha)} \right), \quad (18)$$

conventionally taken as $\alpha(\mu^2 = \Lambda_{\text{TC}}^2) = \alpha_* / [1 + W(e^{-1})] \simeq 0.78 \alpha_*$. The UV and IR behaviors of $\alpha(\mu^2)$ are given by

$$\begin{aligned} \alpha(\mu^2) &\sim \frac{1}{b_0 \ln \frac{\mu}{\Lambda_{\text{TC}}}} \quad (\mu^2 \gg \Lambda_{\text{TC}}^2), \\ &\sim \frac{\alpha_*}{1 + \frac{1}{e} \left(\frac{\mu}{\Lambda_{\text{TC}}} \right)^{b_0 \alpha_*}} \quad (\mu^2 \ll \Lambda_{\text{TC}}^2) \end{aligned} \quad (19)$$

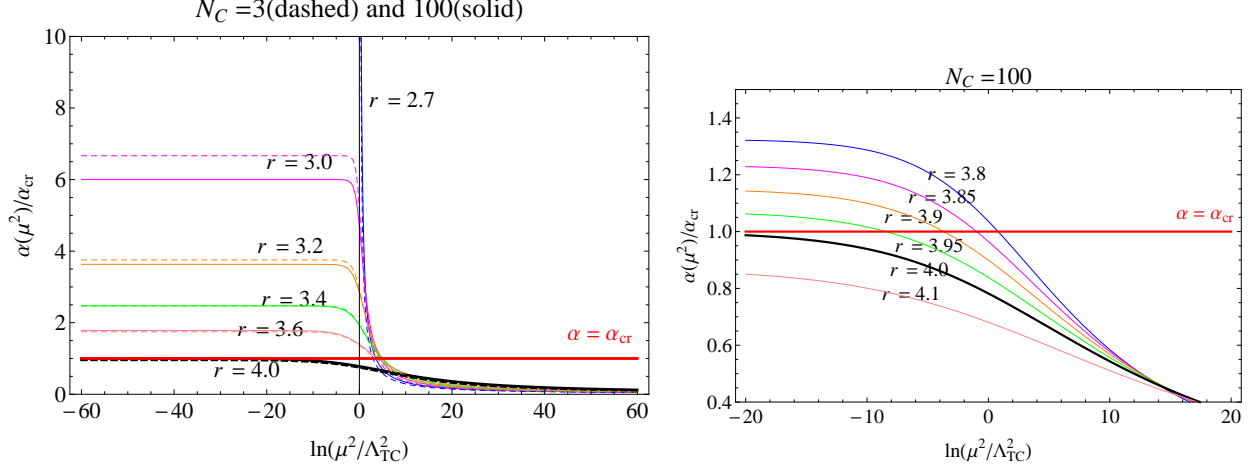


FIG. 2: Two-loop coupling constant $\alpha(\mu^2)$ normalized to the ladder critical coupling $\alpha_{\text{cr}} = \pi/(3C_2)$ at increasing $r = N_F/N_C$ in the anti-Veneziano limit (left). The closed up view near $\alpha = \alpha_{\text{cr}}$ (right).

The intrinsic scale Λ_{TC} is generated by the regularization in the form of the perturbative trace anomaly:

$$\langle \theta_\mu^\mu \rangle^{(\text{perturbative})} = \frac{\beta^{(2\text{-loop})}(\alpha)}{4\alpha} \langle G_{\mu\nu}^2 \rangle \sim -(N_C \alpha) \langle G_{\mu\nu}^2 \rangle \sim -N_F N_C \Lambda_{\text{TC}}^4, \quad (20)$$

where $G_{\mu\nu}$ is the technigluon field strength ^{#9}. Eq.(20) is of course RG invariant: The μ -dependence of $\frac{\beta^{(2\text{-loop})}(\alpha)}{4\alpha} \sim -1/\ln(\mu^2/\Lambda_{\text{TC}}^2)$ is precisely cancelled by that of $\langle G_{\mu\nu}^2 \rangle \sim \Lambda_{\text{TC}}^4 \ln(\mu^2/\Lambda_{\text{TC}}^2)$ in the UV region $\mu^2 > \Lambda_{\text{TC}}^2$ as in the ordinary QCD.

The physics behind the walking/anti-Veneziano limit is very simple: The scale of m_F is determined by the criticality $\alpha(\mu^2 = m_F^2) \sim \alpha_{\text{cr}}$. Let us start with the QCD-like theory with $r_0 \equiv N_F/N_C \sim 1$ where Λ_{TC} is specified as $\alpha(\mu^2 = \Lambda_{\text{TC}}^2) = \mathcal{O}(\alpha_{\text{cr}}) = \mathcal{O}(1/N_C)$, so that we have $m_F = \mathcal{O}(\Lambda_{\text{TC}})$ as in the usual QCD. We then increase $r = r_1 > r_0$, which decreases the coupling mainly in the infrared region $\mu^2 < \Lambda_{\text{TC}}^2$ (biasing infrared-free against asymptotic-free) as a consequence of the increased screening effects of the fermion loop: $\alpha_1(\mu^2) < \alpha_0(\mu^2)$ for $\mu^2 < \Lambda_{\text{TC}}^2$. The criticality $\alpha_1(\mu^2 = m_F^2) = \mathcal{O}(\alpha_{\text{cr}})$ for the infrared-weakened coupling determines the new scale of $(m_F)_{r_1} < (m_F)_{r_0}$. As we continue increasing N_F , we get smaller m_F accordingly, eventually $m_F = 0$ at certain critical $r = r_{\text{cr}} = N_F^{\text{cr}}/N_C$, and the large hierarchy $m_F \ll \Lambda_{\text{TC}}$ is realized near r_{cr} . Beyond that point $r_{\text{cr}} < r < 11/2$, called conformal window, the chiral symmetry is not spontaneously broken, $m_F \equiv 0$. This is depicted in Fig. 2. Then the ladder coupling is regarded as the CBZ IR fixed point in the anti-Veneziano limit:

$$\alpha(x) = \alpha_* \theta(\Lambda_{\text{TC}}^2 - x). \quad (21)$$

B. Solution of the ladder SD equation

Eq.(14) with the ladder coupling Eq.(15) is converted into a differential equation plus IR and UV boundary conditions [62]:

$$(x\Sigma(x))'' + \alpha \frac{3C_2}{4\pi} \frac{\Sigma(x)}{x + \Sigma^2(x)} = 0, \quad (22)$$

$$\lim_{x \rightarrow 0} x^2 \Sigma'(x) = 0, \quad (23)$$

$$(x\Sigma(x))'|_{x=\Lambda^2} = m_0. \quad (24)$$

^{#9} Usual large $N_C (\gg N_F)$ counting would imply $\langle \theta_\mu^\mu \rangle^{(\text{perturbative})} \sim -(N_C \alpha) \langle G_{\mu\nu}^2 \rangle^{(\text{perturbative})} = -\mathcal{O}(N_C^2 \Lambda_{\text{TC}}^4)$ from the gluon loop, which would dominate the fermion-loop of order $-\mathcal{O}(N_C N_F \Lambda_{\text{TC}}^4)$. In the case at hand with $N_F \gg N_C$, however, the fermion-loop dominates instead. See later discussions.

Since Eq.(22) is a nonlinear equation, the absolute value of the $\Sigma(x)$ is determined by the equation itself. In order to have analytical insights, however, we may linearize Eq. (22) by replacing $\Sigma(x)$ in the denominator of the second term in the left-hand side by a constant, m_P . Then the linearized SD equation reads [63]

$$(x\Sigma(x))'' + \alpha \frac{3C_2}{4\pi} \frac{\Sigma(x)}{x + m_P^2} = 0, \quad (25)$$

where the absolute value of $\Sigma(x)$ is determined customarily by

$$m_P \equiv \Sigma(x = m_P^2). \quad (26)$$

A solution of Eq. (25) which satisfies boundary condition Eq. (23) can then be expressed in terms of the hypergeometric function as [63]

$$\Sigma(x) = (\xi m_P) \cdot {}_2F_1\left(\frac{1+\omega}{2}, \frac{1-\omega}{2}, 2, -\frac{x}{m_P^2}\right), \quad (27)$$

where

$$\omega \equiv \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}} \left(\alpha < \alpha_{\text{cr}} = \frac{\pi}{3C_2} \right), \quad i\sqrt{\frac{\alpha}{\alpha_{\text{cr}}} - 1} = i\tilde{\omega} \quad (\alpha > \alpha_{\text{cr}}), \quad (28)$$

and ξ is a numerical coefficient which is determined from the definition of m_P in Eq.(26):

$$\xi = {}_2F_1\left(\frac{1+\omega}{2}, \frac{1-\omega}{2}, 2, -1\right)^{-1} \simeq 1.1 \quad (\omega \simeq 0) \quad 1.0 \quad (\omega \simeq 1). \quad (29)$$

In the limit of $x \gg m_P^2$, the solution can be expanded as

$$\Sigma(x) \simeq \xi m_P \left[\frac{\Gamma(\omega)}{\Gamma(\frac{\omega+1}{2})\Gamma(\frac{\omega+3}{2})} \left(\frac{x}{m_P^2}\right)^{\frac{\omega-1}{2}} + (\omega \leftrightarrow -\omega) \right]. \quad (30)$$

The bare chiral condensate of the technifermion, $\langle \bar{F}F \rangle_0 \equiv \langle \bar{F}_i F_i \rangle_0$ (for a single flavor i with no sum over i), is written in terms of the mass function $\Sigma(x)$ as

$$\langle \bar{F}F \rangle_0 = -\frac{N_C}{4\pi^2} \int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y + \Sigma^2(y)} \quad (31)$$

From Eq.(14) we have

$$\Sigma(\Lambda^2) = m_0 + \frac{3C_2\alpha(\Lambda^2)}{4\pi} \frac{1}{\Lambda^2} \int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y + \Sigma^2(y)}, \quad (32)$$

which yields a formula for the technifermion condensate in terms of the mass function at the cutoff $\Sigma(x = \Lambda^2)$ [64]:

$$\langle \bar{F}F \rangle_0 = \frac{N_C}{3C_2\pi\alpha(\Lambda^2)} \Lambda^2 (m_0 - \Sigma(\Lambda^2)) = \frac{N_C}{\pi^2} \left[\frac{\alpha_{\text{cr}}}{\alpha(\Lambda^2)} \right] [\Lambda^2 (m_0 - \Sigma(\Lambda^2))] = -\frac{N_C\alpha_{\text{cr}}}{\pi^2} \frac{\Sigma'(x)}{\left(\frac{\alpha(x)}{x}\right)'} \Big|_{x=\Lambda^2}. \quad (33)$$

For the nonrunning coupling, the chiral condensate Eq.(33) reads:

$$\langle \bar{F}F \rangle_0 = \frac{N_C}{\pi^2} \left[\frac{\alpha_{\text{cr}}}{\alpha} \right] [\Lambda^4 \cdot \Sigma'(x)|_{x=\Lambda^2}]. \quad (34)$$

C. The conformal phase $\alpha \leq \alpha_{\text{cr}}$

1. The exact massless case: $m_0 \equiv 0$

Let us start with the weak coupling case when the coupling is smaller than the critical coupling, $\alpha = \alpha_* < \alpha_{\text{cr}} = \frac{\pi}{3C_2}$. In the chiral limit $m_0 \equiv 0$, the power-damping solution with Eq.(30) can satisfy the UV boundary condition Eq.(24) only by the trivial solution:

$$\Sigma(p) \equiv 0, \quad \langle \bar{F}F \rangle_0 = 0, \quad (\alpha < \alpha_{\text{cr}}, \quad m_0 \equiv 0). \quad (35)$$

The chiral symmetry is not spontaneously broken, $\langle \bar{F}F \rangle_0 = 0$, producing no mass parameter nor bound states (unparticle phase), in the chiral symmetry limit, even though the scale symmetry is explicitly broken by the intrinsic scale Λ . In this case conformality persists within the ladder approximation, producing no bound states, the situation characteristic to the “conformal phase transition” [45]. This is the explicit example that the theory having intrinsic scale Λ breaking the scale symmetry but has no mass. The same happens e.g., in the NJL model, where the scale symmetry is badly broken by the coupling characterized by the intrinsic scale $G \sim g/\Lambda^{D-2}$ but has no mass in the weak coupling $g < g_{\text{cr}}$.

Although the coupling does not run $\alpha(\mu) \equiv \alpha$ ($\beta(\alpha) \equiv 0$) for $\mu < \Lambda$, there exists the explicit breaking of the scale symmetry due to Λ corresponding to the intrinsic scale Λ_{TC} which is induced quantum mechanically by the regularization. So the scale symmetry is operative only for the energy region $\mu < \Lambda$ (IR conformal). Such an explicit scale-symmetry breaking induced by the regularization manifests itself as the trace anomaly relevant even in the perturbation, see Eq.(20): $\langle \theta_\mu^\mu \rangle^{(\text{perturbative})} = \frac{\beta(\alpha)}{4\alpha(\mu^2)} \langle G_{\nu\lambda}^2(\mu^2) \rangle = -\mathcal{O}(\Lambda^4)$. Accordingly, there exists no extra scale and so does no nonperturbative trace anomaly:

$$\langle \theta_\mu^\mu \rangle^{(NP)} \equiv \langle \theta_\mu^\mu \rangle^{(\text{full})} - \langle \theta_\mu^\mu \rangle^{(\text{perturbative})} = 0 \quad (\alpha < \alpha_c). \quad (36)$$

2. Small explicit breaking: $m_0(\neq 0) \ll \Lambda$

If we introduce the explicit fermion mass $m_0 = m_0(\Lambda^2) \neq 0$ which is another source of the explicit breaking of the scale symmetry in addition to $\Lambda(\gg m_0)$, then the exact IR conformality is gone and physical states including the bound states can appear, with the generic mass parameter M solely due to $m_0 \neq 0$, all of which are obeying the typical hyperscaling relation [65],

$$M \sim \Lambda \left(\frac{m_0}{\Lambda} \right)^{\frac{1}{1+\gamma_m}}, \quad \text{or} \quad m_0 \sim M \left(\frac{M}{\Lambda} \right)^{\gamma_m}, \quad (37)$$

where the γ_m is the mass anomalous dimension. If M is the renormalized mass of the fermion m_R , Eq.(37) takes the conventional form: $m_0 = Z_m m_R$, with the renormalization constant $Z_m = (m_R/\Lambda)^{\gamma_m}$.

In fact a nontrivial solution of the ladder SD equation, Eq.(30), satisfies the UV boundary condition Eq.(24):

$$m_0 = \xi m_P \left[\frac{\Gamma(\omega)}{\Gamma(\frac{\omega+1}{2})^2} \left(\frac{\Lambda^2}{m_P^2} \right)^{\frac{\omega-1}{2}} + (\omega \leftrightarrow -\omega) \right], \quad (38)$$

where $m_P = m_R$ is now the renormalized mass (or current mass) due to this explicit scale breaking mass m_0 , with [24]

$$Z_m \equiv \frac{m_0}{m_R} = \xi \left[\frac{\Gamma(\omega)}{\Gamma(\frac{\omega+1}{2})^2} \left(\frac{\Lambda^2}{m_R^2} \right)^{\frac{\omega-1}{2}} + (\omega \leftrightarrow -\omega) \right], \quad (39)$$

or we have [48]

$$\gamma_m = \lim_{m_R/\Lambda \rightarrow 0} \frac{\partial \log Z_m}{\partial \log(m_R/\Lambda)} = 1 - \omega = 1 - \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}}, \quad (\alpha < \alpha_{\text{cr}}). \quad (40)$$

For $\alpha \ll 1$ ($\omega \simeq 1$) it coincides with the perturbative one $\gamma_m \simeq \frac{\alpha}{2\alpha_{\text{cr}}} = \frac{3C_2\alpha}{2\pi} \simeq A/\ln(\Lambda^2/m_R^2)$ and $Z_m \simeq (\ln(\Lambda/m_R))^{-A/2}$, with $A \simeq 18C_2/(11N_C - 2N_F)$. For $\alpha \rightarrow \alpha_{\text{cr}}$ ($\omega \rightarrow 0$), on the other hand, we have $\gamma_m \rightarrow 1$ and $Z_m \rightarrow \frac{2\xi}{\pi} \frac{m_R}{\Lambda}$.

The asymptotic solution Eq.(30) takes the form

$$\Sigma(x) \sim m_R \left(\frac{x}{m_R^2} \right)^{-\gamma_m/2} \quad (\alpha < \alpha_{\text{cr}}), \quad (41)$$

which is consistent with the Operator Product Expansion (OPE). Such a nonzero running mass is a genuine effect of the nonperturbative dynamics of the ladder SD equation having a set of particular all order diagrams in the conformal phase $\alpha < \alpha_{\text{cr}}$ without SSB of the chiral symmetry. Accordingly, the beta function after including the $m_R \neq 0$ effects would no longer be a constant, although the ladder coupling as a input is treated as a constant: $\beta^{(\text{ladder})}(\alpha) = 0$.

Note that $m_0 = m_0(\Lambda) \rightarrow 0$ as $\Lambda \rightarrow \infty$. Here we mention that the cutoff Λ plays a crucial role to identify the solution of the SD equation [66], whether it is a spontaneously broken solution or explicitly broken one: The spontaneous chiral symmetry breaking solution with $\Sigma(x) \neq 0$ for $m_0 \equiv 0$ exists only for the strong coupling $\alpha > \alpha_{\text{cr}} = \pi/(3C_2)$ in the presence of the cutoff $\Lambda < \infty$, while for weak coupling $\alpha < \alpha_{\text{cr}}$ there exists only the explicit chiral symmetry breaking solution such that $\Sigma(x) \neq 0$ for $m_0 \neq 0$ and $\Lambda < \infty$, with $m_0 \rightarrow 0$ while the renormalized mass $m_R \neq 0$ for $\Lambda \rightarrow \infty$. The explicit breaking solution would be confused with the spontaneous breaking, if we took (erroneously) $\Lambda \rightarrow \infty$ from the onset in the SD equation [67]. See the discussions in Ref. [66].

D. The SSB phase $\alpha > \alpha_{\text{cr}}$

Now we discuss the strong coupling phase, $\alpha > \alpha_{\text{cr}} = \frac{\pi}{3C_2}$ and $m_0 \equiv 0$, where the nontrivial solution $\Sigma(x) \neq 0$ exists even at $m_0 \equiv 0$, that is, the chiral symmetry is spontaneously broken, i.e., $\langle \bar{F}F \rangle_0 \neq 0$. The SSB solution $\Sigma(x)$ in Eq.(27) with $\omega = i\tilde{\omega}$ in Eq.(28) takes the oscillating form [24, 62, 66]

$$\Sigma(x) \simeq \xi \frac{m_F^2}{\sqrt{x}} \sqrt{\frac{8 \text{cth} \frac{\pi \tilde{\omega}}{2}}{\pi \tilde{\omega}(\tilde{\omega}^2 + 1)}} \sin \left(\frac{\tilde{\omega}}{2} \ln \left(\frac{16x}{m_F^2} \right) - \tilde{\omega} \right) \quad (x \gg m_F^2), \quad \tilde{\omega} = \left(\frac{\alpha}{\alpha_{\text{cr}}} - 1 \right)^{1/2}, \quad (42)$$

where we set the dynamical mass m_F as $m_P = m_F$ such that $\Sigma(x = m_F^2) = m_F$, and $\xi = F(1/2, 1/2, 2; -1)^{-1} \simeq 1.1$. The oscillating solution can satisfy the UV boundary condition Eq.(24) for $m_0 = 0$:

$$\begin{aligned} 0 = m_0 &\simeq (x\Sigma(x))'|_{x=\Lambda^2} = \xi \frac{m_F^2}{\Lambda} \sqrt{\frac{8 \text{cth} \frac{\pi \tilde{\omega}}{2}}{\pi \tilde{\omega}}} \sin \left(\frac{\tilde{\omega}}{2} \ln \frac{16\Lambda^2}{m_F^2} - \tilde{\omega} + \tan^{-1}(\tilde{\omega}) \right) \\ &\simeq \frac{4\xi}{\pi \tilde{\omega}} \frac{m_F^2}{\Lambda} \sin \left(\frac{\tilde{\omega}}{2} \ln \frac{16\Lambda^2}{m_F^2} \right) \end{aligned} \quad (43)$$

which is fulfilled by the vanishing phase, $\frac{\tilde{\omega}}{2} \ln \left(\frac{16\Lambda^2}{m_F^2} \right) \simeq n\pi$ ($n = 1, 2, 3, \dots$), with $n = 1$ being the ground state [24]:

$$m_F \simeq 4\Lambda \cdot \exp \left(-\frac{\pi}{\sqrt{\frac{\alpha}{\alpha_{\text{cr}}} - 1}} \right) \quad (\alpha \gtrsim \alpha_{\text{cr}} = \frac{\pi}{3C_2} = \frac{\pi}{3} \frac{2N_C}{N_C^2 - 1}). \quad (44)$$

Then the technifermion acquires the dynamical mass m_F in an essential-singularity (non-analytic) form (Miransky scaling, or the BKT transition) which implies a large hierarchy $m_F \ll \Lambda$ for $\alpha \simeq \alpha_{\text{cr}}$, where the cutoff Λ as a regulator may be regarded as the intrinsic scale Λ_{TC} .

1. Nonperturbative running (walking) coupling, with the IR fixed point as a UV fixed point

As we already mentioned in the Introduction, the Miransky-BKT scaling can create a large hierarchy, “criticality hierarchy”, $m_F \ll \Lambda = \Lambda_{\text{TC}}$ for $\alpha \simeq \alpha_* \simeq \alpha_{\text{cr}}$, which dictates that the coupling no longer constant but does depend on the Λ/m_F as in Eq.(44), in such a way that the scale symmetry still remains approximately as the coupling is walking $\alpha(\mu^2) \simeq \text{constant}$ for the wide region $m_F^2 \ll \mu^2 < \Lambda_{\text{TC}}^2$ as shown in Eq.(4):

$$\begin{aligned} \beta^{(NP)}(\alpha) &= \Lambda \frac{\partial \alpha(\Lambda)}{\partial \Lambda} = -\frac{2\pi^2 \alpha_{\text{cr}}}{\ln^3 \left(\frac{4\Lambda}{m_F} \right)} \Rightarrow -\frac{2\alpha}{\pi} \left(\frac{\alpha}{\alpha_{\text{cr}}} - 1 \right)^{\frac{3}{2}} \approx 0 \quad (< 0), \\ \alpha(\mu) &= \alpha_{\text{cr}} \left(1 + \frac{\pi^2}{\ln^2 \left(\frac{\mu}{\mu_{\text{IR}}} \right)} \right) \approx \alpha_{\text{cr}}, \end{aligned} \quad (45)$$

even when the perturbative coupling (input coupling) is nonrunning, $\beta(\alpha)|^{\text{perturbative}} \equiv 0$. Here $\mu_{\text{IR}} (\sim m_F/4)$ is given as $\ln(\mu/\mu_{\text{IR}}) \simeq \ln(4\mu/m_F)[1 + \pi^2/\ln^2(4\mu/m_F)]^{-1}$ ^{#10}.

^{#10} Solution of $\frac{\partial \alpha}{\partial \ln \mu} = \beta^{(NP)}(\alpha)$ is $\frac{1}{\pi} \ln \mu = \left(\frac{\alpha}{\alpha_{\text{cr}}} - 1 \right)^{-1/2} + \tan^{-1} \left(\frac{\alpha}{\alpha_{\text{cr}}} - 1 \right)^{1/2} \simeq \frac{\alpha}{\alpha_{\text{cr}}} \left(\frac{\alpha}{\alpha_{\text{cr}}} - 1 \right)^{-1/2}$. Or, $\alpha(\mu) \simeq \alpha_{\text{cr}} \left(1 + \frac{\pi^2}{\ln^2 \mu} \left[1 + \frac{\pi^2}{\ln^2 \mu} \right]^2 \right)$.

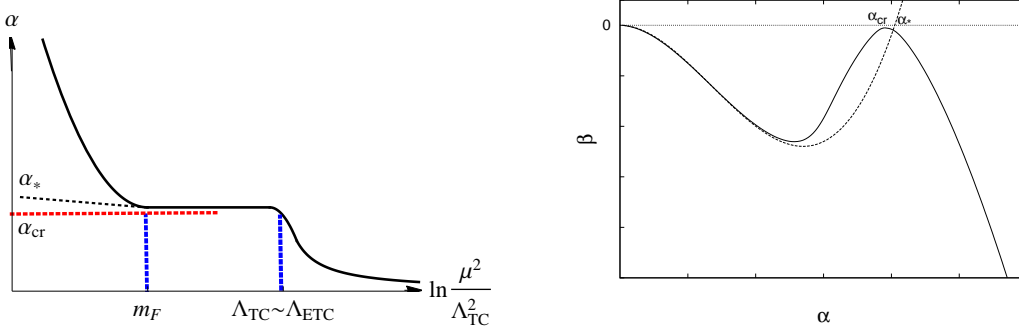


FIG. 3: Possible perturbative running coupling (left) and the beta function (right) in the region $\alpha < \alpha_{\text{cr}}$, in comparison with the nonperturbative region $\alpha > \alpha_{\text{cr}}$.

Note [18, 47] that the form of the beta function in Eq.(45) for $\alpha > \alpha_{\text{cr}}$ has a multiple zero, and is in fact not Taylor-expandable, with $\frac{d\beta(\alpha)}{d\alpha}|_{\alpha=\alpha_{\text{cr}}} = 0$ (without linear zero term), $|\frac{d^n\beta(\alpha)}{d\alpha^n}|_{\alpha=\alpha_{\text{cr}}} = \infty$ ($n \geq 2$), reflecting the conformal phase transition of the Miransky-BKT essential singularity scaling. This is in sharp contrast to the two-loop beta function Eq.(16) having a Taylor expansion with the first term of the linear zero at $\alpha = \alpha_*$: $\beta^{(\text{perturbative})} \sim \alpha - \alpha_* + \mathcal{O}((\alpha - \alpha_*)^2)$. Such a perturbative IR zero makes sense only for $\alpha(\mu) \ll \alpha_* \lesssim \alpha_{\text{cr}}$ (deep conformal phase). Since the beta function should be continuous across the critical point α_{cr} , it should be continued to the conformal phase $\alpha < \alpha_{\text{cr}}$ with zero curvature. In the broken phase $\alpha_{\text{cr}} < \alpha_*$ where α_* is washed out, the two-loop beta function is operative only for $\alpha(\mu) < \alpha_{\text{cr}}$ in the far-ultraviolet region $\mu > \Lambda_{\text{TC}}$, where the dynamics is irrelevant to the electroweak symmetry breaking.

Since the critical coupling α_{cr} behaves as the UV fixed point, the original ladder coupling as an ideal limit of the IR fixed point (viewed from the UV region $\mu^2 > \Lambda_{\text{TC}}^2$) in the anti-Veneziano limit may be identified with the UV fixed point viewed from the IR side $\mu^2 < \Lambda_{\text{TC}}^2$. Then the effective coupling $N_C\alpha(\mu^2)$ keeps strong in IR region all the way up to the intrinsic scale Λ_{TC} so that the anomalous dimension is very large in that region. Now the would-be CBZ IR fixed point $\alpha \simeq \alpha_* \simeq \alpha_c$ is regarded as the UV fixed point of the nonperturbative running (walking) coupling $\alpha(\mu) \approx \alpha_c$ in the wide IR region $m_F < \mu < \Lambda = \Lambda_{\text{TC}}$ for the characteristic large hierarchy $m_F \ll \Lambda_{\text{TC}}$. See Fig.3 [47, 49] (See also Ref.[50] for a similar observation.). This is the essence of the WTC. The new scale m_F (denoted as Λ_{TC} in Ref. [6], which should not be confused with Λ_{TC} in this paper) is regarded as the second RG-independent quantity as,

$$m_F = 4\mu \cdot \exp\left(-\int^{\alpha(\mu)} \frac{d\alpha}{\beta^{(NP)}(\alpha)}\right) \simeq 4\Lambda_{\text{TC}} \cdot \exp\left(-\frac{\pi}{\sqrt{\frac{\alpha(\Lambda_{\text{TC}})}{\alpha_c} - 1}}\right) \ll \Lambda_{\text{TC}}, \quad (46)$$

with $\beta^{(NP)}(\alpha)$ given in Eq.(4). Compare it with Eq.(18).

On the other hand, in the UV region $\mu > \Lambda_{\text{TC}}$ ($\alpha(\mu) < \alpha_c \simeq \alpha_*$), the coupling runs as the usual perturbative asymptotically free theory: $\alpha(\mu) \sim 1/\ln\mu$. See Fig.3. Such a perturbative region $\alpha < \alpha_c$ is actually irrelevant to the physics of WTC, since the theory is expected to become only a part of more fundamental (unified) theory including the SM sector, say, the ETC [41] or technicolored composite model [15].

Incidentally, the original setting of the WTC [6] was an asymptotically non-free theory with small perturbative beta function $1 \gg \beta(\alpha) > 0$, as in the technicolored preon model [15] where the technifermions as well as the quarks and leptons are composites on the same footing and the technicolor gauge at composite level is asymptotically non-free in the perturbative sense due to the formation of many composite technifermions (though the technicolor at the preon level is asymptotically free). This perturbative setting makes sense only in the weak coupling phase $\alpha < \alpha_c$. On the other hand, in the strong coupling phase $\alpha > \alpha_{\text{cr}}$ ($\mu < \Lambda = \Lambda_{\text{TC}} \sim \Lambda_{\text{ETC}}$ (or $\Lambda_{\text{composite}}$), both the asymptotically-free theories with the CBZ IR fixed point and the asymptotically non-free theories yield the same nonperturbative beta function Eq.(4), i.e., Eq.(5) and Fig.1(a) of Ref. [6], having a UV fixed point at $\alpha = \alpha_c$, which is only the physical issue of the WTC. In fact, in the asymptotically non-free theory with the perturbative coupling growing function of μ in units of the Landau pole $\Lambda = \Lambda_{\text{Landau}} = \Lambda_{\text{Composite}}$, the ladder SD equation tells us that the dynamical mass m_F is generated as a scale when the coupling exceeds the critical coupling $\alpha(\mu = m_F) > \alpha_{\text{cr}}$. Then, in contrast to the infrared-free phase $\alpha < \alpha_{\text{cr}}$ of the asymptotically-free theory (Coulomb phase), the physics

in the strong coupling phase is precisely the same as the WTC in the anti-Veneziano limit of the asymptotically free theory, with only exception being that the Λ in the Miransky scaling Eq.(2) should now read the Landau pole scale $\Lambda_{\text{Landau}} = \Lambda_{\text{Composite}}$ (“compositeness condition” [5], to be identified with the composite scale in the technicolored preon model [15], to generate the effective four-fermion interactions in Eq.(50)) instead of the intrinsic scale of the asymptotically-free theory. From the model building point of view, it does not make sense [64] whether the WTC in isolation is asymptotically free or nonfree in the region, $\alpha < \alpha_{\text{cr}}$ ($\mu > \Lambda = \Lambda_{\text{TC}} \sim \Lambda_{\text{ETC}}(\Lambda_{\text{Composite}})$), where the theory is already changed into a more fundamental unified theory, ETC or preon theory both being asymptotically-free anyway.

2. Large anomalous dimension $\gamma_m = 1$ and enhanced chiral condensate

Eq.(42) together with Eq.(43) yields the asymptotic form of $\Sigma(x)$ at $m_F^2 \ll x \lesssim \Lambda^2 = \frac{m_F^2}{16} \exp(\frac{2\pi}{\tilde{\omega}})$ [6]:

$$\Sigma(x) \simeq \xi \frac{m_F^2}{\sqrt{x}} \frac{4}{\pi \tilde{\omega}} \sin\left(\frac{\tilde{\omega}}{2} \ln(16x/m_F^2) - \tilde{\omega}\right) \simeq \xi \frac{m_F^2}{\sqrt{x}} \frac{4}{\pi \tilde{\omega}} \sin(\pi - \tilde{\omega}) \simeq \frac{4\xi}{\pi} \frac{m_F^3}{x} \left(\frac{x}{m_F^2}\right)^{1/2}, \quad (47)$$

where the logarithmic x -dependence is absent for the region $\frac{\tilde{\omega}}{2} \ln(16x/m_F^2) \sim \frac{\tilde{\omega}}{2} \ln(16\Lambda^2/m_F^2) \simeq \pi$. In the proposal of the WTC [6, 7], this asymptotic form $\Sigma(x) \sim \frac{m_F^3}{x} \left(\frac{x}{m_F^2}\right)^{1/2}$ was identified with the OPE of $\Sigma(x)$ at $m_F^2 \ll x \lesssim \Lambda^2$ #11:

$$\Sigma(x) \sim \frac{m_F^3}{x} \left(\frac{x}{m_F^2}\right)^{\gamma_m/2}, \quad (48)$$

to conclude a large anomalous dimension in the SSB phase near criticality (UV fixed point) [6]:

$$\gamma_m = 1 \quad \left(\tilde{\omega} = \sqrt{\frac{\alpha}{\alpha_{\text{cr}}} - 1} = \frac{\pi}{\ln \frac{4\Lambda}{m_F}} \simeq 0 \right). \quad (49)$$

The large anomalous dimension $\gamma_m = 1$ in the SSB phase was also compared with the anomalous dimension Eq.(40) in the conformal phase ($\alpha < \alpha_{\text{cr}}$) at criticality: $\gamma_m = 1 - \sqrt{1 - \alpha/\alpha_{\text{cr}}} \rightarrow 1$ ($\alpha \rightarrow \alpha_{\text{cr}} - 0$) (see Eqs.(6) and (7) and Fig. 1(b) of Ref. [6]).

The ladder result, $\gamma_m = 1$, in Eq.(49) is a direct consequence of the scale-symmetric strong dynamics first found in the ladder SD equation in the proposal of WTC [6] as a solution of the FCNC problem of the original TC as a simple scale-up of the QCD [16]. Before advent of the WTC, a large anomalous dimension of the TC dynamics $\gamma_m \gtrsim 1$ was anticipated [13] (see also [14, 15]) to enhance the bare condensate by the factor $Z_m^{-1} = (\Lambda/m_F)^{\gamma_m}$, as a solution of the FCNC problem, based on the pure assumption of the UV fixed point at strong coupling.

Masses of the quarks/leptons are generated through communication between quarks/leptons ψ and the technifermions F through extra dynamics such as the ETC [41], or the technicolored preon model [15] (quarks, leptons and technifermions are composites on the same footing), which generically give effective four-fermion interactions:

$$G_a (\bar{\psi}\psi)^2, \frac{G_b}{N_C} (\bar{F}F)^2, \frac{G_c}{N_C} (\bar{\psi}\psi) (\bar{F}F), \quad (50)$$

where the three types of four-fermion couplings $G_{a,b,c} = \mathcal{O}\left(\frac{a,b,c}{\Lambda^2}\right)$ are on the same order of magnitude characterized by the scale of the extra dynamics $\Lambda = \Lambda_{\text{ETC}} \sim \Lambda_{\text{TC}}$, except for the numerical factors $a, b, c = \mathcal{O}(1)$ depending on the explicit model, and the factor $1/N_C$ for G_b, G_c is the effect of the Fierz transformation from the current \times current four-fermion coupling from the ETC gauge exchanges. While G_a yields FCNC, G_c yields the quark/lepton mass:

$$m_{q/l} = -\frac{G_c}{N_C} \langle \bar{F}F \rangle_0 \sim -\frac{c Z_m^{-1}}{\Lambda^2} \frac{\langle \bar{F}F \rangle_R}{N_C} \simeq c \frac{m_F^2}{\Lambda}, \quad Z_m^{-1} \sim \frac{\Lambda}{m_F} \quad (51)$$

#11 The ladder SD solution with respect to OPE was also discussed in [68] in a way somewhat different than Refs. [6, 7], concerning the logarithmic dependence. The log peculiarity is just on the point $\alpha \equiv \alpha_{\text{cr}}$ where no SSB takes place. Absence of log in the SSB phase is consistently seen in Eq.(53) and Eq.(54). See also the OPE, Eq.(56).

where $\langle \bar{F}F \rangle_R / N_C = -\mathcal{O}(m_F^3) = -\mathcal{O}(\frac{1}{3} \text{ TeV})^3$ is the condensate renormalized at $\mu = m_F$, and $\Lambda_{\text{ETC}} \sim 10^3 \text{ TeV}$, thus arriving at the typical order of quarks/leptons mass (except for the top quark) : $m_{q/l} \sim 0.1 \text{ GeV}$. This is in sharp contrast to the ordinary QCD with $Z_m^{-1} \sim (\ln(\Lambda/m_F))^{A/2} = \mathcal{O}(1)$ and $\gamma_m \simeq \frac{3C_2\alpha}{2\pi} \simeq A/\ln(\Lambda^2/m_F^2) \approx 0$ with $A = 18C_2/(11N_C - 2N_F)(< 1)$:

$$m_{q/l} = G_c \langle \bar{F}F \rangle_0 \sim \frac{c}{\Lambda^2} \langle \bar{F}F \rangle_R \sim 0.1 \text{ MeV}, \quad \langle \bar{F}F \rangle_0 = \mathcal{O}(\langle \bar{F}F \rangle_R). \quad (52)$$

In order to keep track of the concrete analytical expression of the ladder results (in a linearized version Eq.(25)), we here list results of the precise (linearized) ladder computation of the chiral condensate $\langle \bar{F}F \rangle_0$, using the explicit form of the SSB solution Eq.(42), based on Eq.(33) and/or (34). (For details see Appendix A.)

The bare condensate and the mass renormalization constant $Z_m = m_0/m_R$ take the form in agreement with Ref. [69]:

$$\langle \bar{F}F \rangle_0 = -\frac{\xi N_C}{\pi^2} \frac{\alpha_{\text{cr}}}{\alpha(\Lambda^2)} \Lambda^2 \Sigma(\Lambda) \simeq -\frac{4\xi N_C}{\pi^3} m_F^2 \Lambda, \quad (53)$$

$$Z_m = \frac{m_0}{m_R} \simeq \frac{2\xi}{\pi} \frac{m_F}{\Lambda}, \quad (54)$$

$$\langle \bar{F}F \rangle_R = Z_m \langle \bar{F}F \rangle_0 \simeq -\frac{8\xi^2 N_C}{\pi^4} m_F^3. \quad (55)$$

Thus the asymptotic form of $\Sigma(x)$ ($m_0 \neq 0$) in Eq.(42) with $m_P = \Sigma(x = m_P^2) \simeq m_F + m_R$ ($m_R \ll m_F$) is perfectly consistent with the OPE for x such that $\tilde{\omega} \ln\left(\frac{16x}{m_F^2}\right) \simeq \pi$:

$$\begin{aligned} \Sigma(x) &\sim \frac{4\xi}{\pi} \frac{1}{\tilde{\omega}} \frac{m_P^2}{\sqrt{x}} \sin\left(\frac{\tilde{\omega}}{2} \ln\left(\frac{16x}{m_P^2}\right) - \tilde{\omega}\right) \simeq \frac{4\xi}{\pi} \frac{1}{\tilde{\omega}} \frac{m_P^2}{\sqrt{x}} \sin\left(\frac{\tilde{\omega}}{2} \ln\left(\frac{16x}{m_F^2}\right) - \tilde{\omega} \ln\left(1 - \frac{m_R}{m_F}\right) - \tilde{\omega}\right) \\ &\simeq \frac{4\xi}{\pi} \left[\frac{m_F m_R}{\sqrt{x}} + \frac{m_F^2}{\sqrt{x}} \right] \\ &\sim \frac{4\xi}{\pi} m_R \left(\frac{x}{m_F^2} \right)^{-\gamma_m/2} - \frac{\pi^3}{2\xi N_C} \frac{\langle (\bar{F}F)_R \rangle}{x} \left(\frac{x}{m_F^2} \right)^{\gamma_m/2}. \end{aligned} \quad (56)$$

Combining Eq.(53) with the Pagels-Stokar formula Eq.(B4), $F_\pi^2 = \frac{N_C \xi^2}{2\pi^2} m_F^2$, or $m_F^2 \simeq \frac{4\pi^2}{\xi^2} \frac{1}{N_F N_C} v_{\text{EW}}^2$, we have for $\Lambda = \Lambda_{\text{TC}} \sim \Lambda_{\text{ETC}}$:

$$m_{q/l} = \frac{c}{N_C} \frac{\langle \bar{F}F \rangle_0}{\Lambda_{\text{ETC}}^2} = y^{\text{eff}} \cdot v_{\text{EW}}, \quad y^{\text{eff}} = \frac{c}{N_F N_C} \mathcal{O}\left(\frac{4}{\xi\pi} \frac{4v_{\text{EW}}}{\Lambda_{\text{ETC}}}\right) = \mathcal{O}(10^{-3}). \quad (57)$$

3. Large Anomalous dimension and amplification of the symmetry violation

A striking feature of the WTC having the large anomalous dimension $\gamma_m = 1$ is that the explicit symmetry breaking by a small Lagrangian parameter is enhanced by the strong dynamics near the criticality being persistent all the way up to the intrinsic scale Λ_{TC} . The quark/lepton mass enhancement already discussed is a typical such example: Such masses come from formally the small explicit breaking of the SM fermion chiral symmetry by the small ETC gauge coupling, g_{ETC} , leading to the small ETC-induced four-fermion coupling $G_c \sim g_{\text{ETC}}^2/M_{\text{ETC}}^2 \sim c/\Lambda_{\text{ETC}}^2 (\ll 1/m_F^2)$, where $M_{\text{ETC}} \sim g_{\text{ETC}}\Lambda_{\text{ETC}}$ is the ETC gauge boson mass generated by the SSB of the ETC gauge symmetry down to the WTC, with the order parameter v_{ETC} of the SSB of the ETC gauge symmetry being $v_{\text{ETC}} \sim \Lambda_{\text{ETC}}$. Though the coupling is small, the resultant mass is amplified by the walking dynamics with $Z_m^{-1} \simeq \Lambda/m_F > 10^3$, as we discussed in the above.

Here we briefly comment on yet another quantity subject to this enhancement effects due to the large anomalous dimension. It is the technipions mass, another phenomenological issue of the generic WTC. The technipions are the left-over (pseudo) NG bosons besides the (fictitious) NG bosons absorbed into SM gauge bosons. They exist in a large class of the WTC having large $N_F (> 2)$ and will be a smoking gun of this class of WTC in the future LHC.

Technipion mass is all from explicit breaking outside of the WTC sector, i.e, SM gauge interactions and ETC gauge interactions (G_b terms in Eq.(50)): The estimation of the masses of the technipions in the WTC is done, based on

the first order perturbation of the explicit chiral symmetry breaking by the “weak gauge couplings” of SM gauge interactions and the ETC gauge interactions (Dashen’s formula),

$$M_\pi^2(\text{SM}) \sim \frac{C_2^{\text{SM}} \alpha_{\text{SM}}}{4\pi F_\pi^2} \int dx (\Pi_V(x)_V - \Pi_A(x)) \quad (58)$$

$$M_\pi^2(\text{ETC}) \sim \frac{1}{F_\pi^2} \frac{\alpha_{\text{ETC}}}{M_{\text{ETC}}^2} \langle \frac{1}{N_C} (\bar{F}F) (\bar{F}F) \rangle_0 \sim \frac{1}{F_\pi^2} \frac{b}{\Lambda^2} \frac{1}{N_C} (\langle \bar{F}F \rangle_0)^2 \quad (59)$$

up to Clebsch-Gordan coefficient depending on the detailed model, where $\Pi_{V,A}(x)$ are current correlators of vector and axialvector currents. This is the same strategy as the QCD estimate of the $\pi^+ - \pi^0$ mass difference, where the explicit chiral symmetry breaking is given by the QED lowest order coupling, while the full QCD nonperturbative contributions are estimated through the current correlators by various method like ladder, holography, lattice, etc..

It is obvious that $M_\pi^2(\text{ETC})$ is enhanced through the condensate by the anomalous dimension as $(Z_m^{-1})^2 \sim (\Lambda/m_F)^{2\gamma_m}$, as was noted before the advent of the WTC [13, 15], and was confirmed in the WTC with $\gamma_m = 1$ based on the concrete scale-invariant dynamics, the ladder SD equation [6]. $M_\pi^2(\text{SM})$ is also enhanced by the large anomalous dimension $\gamma_m = 1$ [70], since the high energy behavior is slower damping by the anomalous dimension $\Pi_V(x) - \Pi_A(x)|_{x>m_F^2} \sim \alpha(x) \frac{(\bar{F}F)_R^2}{x^2} \left(\frac{x}{m_F^2}\right)^{\gamma_m} \sim \frac{N_C m_F^4}{x}$ (A similar observation was made without notion of the anomalous dimension [71]). Then we have a large mass for the technipions [32, 70, 72]:

$$M_\pi^2(\text{ETC}) \sim 2\pi^2 b m_F^2 = \mathcal{O}((\text{TeV})^2), \quad [M_\pi^2(\text{SM})]_{x>m_F^2} \sim (C_2^{\text{SM}} \alpha_{\text{SM}}) m_F^2 \ln(\Lambda^2/m_F^2) \lesssim (\text{TeV})^2, \quad (60)$$

where the Pagels-Stokar formula Eq.(B1) is used ^{#12}.

Striking fact is that although the explicit chiral symmetry breakings are formally very small due to the “weak gauge couplings”, the nonperturbative contributions from the WTC sector lift all the technipions masses to the TeV region so that they all lose the nature of the “pseudo NG bosons”. This is actually a universal feature of the dynamics with large anomalous dimension, “amplification of the symmetry violation” [8], as dramatically shown in the top quark condensate model [3], based on the gauged NJL model with large anomalous dimension $\gamma_m \simeq 2$ [9].

This amplification effect should not be confused with that of the pseudo NG boson mass due to the technifermion bare mass effects, like the pion mass due to the current quark mass, $F_\pi^2 m_\pi^2 = 2m_0 \langle \bar{\psi}\psi \rangle_0 = 2m_R \langle \bar{\psi}\psi \rangle_R$, which are not amplified by the large anomalous dimension, since the bare mass operator as the explicit breaking is the RG invariant, $m_0(\bar{F}F)_0 = m_R(\bar{F}F)_R$, and hence is ignorant about the anomalous dimension within the WTC sector. In the actual technicolor model, all the technifermions are set to be exactly massless and such a type of explicit breaking is not considered anyway.

Note that although the left-over light spectra are just three exact NG bosons absorbed into W/Z bosons, our theory with $N_F \gg 2$ in the anti-Veneziano limit is completely different from the model with massless flavors $N_f = 2$ where the symmetry breaking is $SU(2)_L \times SU(2)_R/SU(2)_V$. In fact even though all the NG bosons, other than the three exact NG bosons to be absorbed into W, Z bosons, are massive and decoupled from the low energy physics, they are composite of the linear combinations of all the N_F technifermions not just 2 of them.

In fact, the technifermions do not acquire the explicit mass from these explicit breaking terms, and hence the walking behavior of the coupling of the large N_F in the anti-Veneziano limit is not drastically changed. They actually get some effects on the dynamical masses, as a result of the vacuum alignment including the extra gauge interactions, which are to be treated as the corrections to the ladder SD equation including not only the WTC gauge coupling but also the SM gauge interactions, with the modified criticality $C_2^{\text{WTC}} \alpha^{\text{WTC}} + C_2^{\text{SM}} \alpha^{\text{SM}} > \frac{\pi}{3}$, and the ETC gauge interactions as corrections to the WTC gauge interaction in the ladder kernel in a form of the four-fermion couplings $G_{b,c}$ in Eq.(50).

While G_c is in general (except for the top quark) a small feedback of the SM fermion condensate to the technifermion condensate in the coupled SD equation, G_b is potentially strong effects on the phase structure in a way that the critical coupling is replaced by the critical line (surface) of the two-dimensional (multi-dimensional) coupling space, (α, g) ,

^{#12} Note that $M_\pi^2(\text{SM})$ has also IR contributions from $x \lesssim m_F^2$, which is less than the UV contributions as far as the (techni-sector) S parameter is large $S > 0.3$, thus $M_\pi(\text{SM})^2 = [M_\pi^2(\text{SM})]_{x>m_F^2} + [M_\pi^2(\text{SM})]_{x<m_F^2} < \mathcal{O}((1.5\text{TeV})^2)$, in somewhat tension with the present LHC limit for the colored technipions. (The techni-sector S parameter can be cancelled by the ETC sector contribution to be consistent with the S parameter value constrained by the precision experiments.) A possible way out besides the ETC cancellation would be the strong gluon condensate which has not been incorporated into the ladder SD approach but has been shown in the holography to dramatically enhance the infrared part $[M_\pi^2(\text{SM})]_{x<m_F^2}$, in accord with the suppression of the techni-sector S parameter $S < 0.1$ [32].

This gluonic effect enhances $M_\pi^2(\text{ETC})$.

with $g = \frac{N_C}{4\pi^2} \Lambda^2 G_b$ in the gauged NJL model [10, 11], as analyzed with the kernel having extra contributions of the SM (running) gauge couplings and ETC-induced four-fermion interaction (strong ETC technicolor). In that case, the SSB solution of the SD equation exists even for the weak gauge coupling $\alpha < \alpha_{\text{cr}}$ because of the additional strong NJL four-fermion coupling ($\alpha \rightarrow 0$ is the NJL limit)^{#13}. The result shows drastic effects with the anomalous dimension even larger, $1 < \gamma_m = 1 + \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}} < 2$ [9] at the critical line. This is particularly useful for reproducing the top quark mass which would need more enhancement than other quarks due to such a large anomalous dimension [9, 73]. See the discussions in the last section.

In fact, the SSB solution takes the form instead of the Miransky scaling:

$$m_F^{2\omega} = \Lambda^{2\omega} \left(\frac{g - g_{\text{cr}}^{(+)}}{g - g_{\text{cr}}^{(-)}} \right), \quad g_{\text{cr}}^{(\pm)} = \frac{1}{4} (1 \pm \omega)^2, \quad \omega \equiv \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}} \quad (0 < \alpha < \alpha_{\text{cr}}), \quad (61)$$

and the anomalous dimension [9, 12]:

$$\gamma_m = 2g + \frac{\alpha}{2\alpha_{\text{cr}}}, \quad \gamma_m^{(\pm)} = \gamma_m \Big|_{g=g_{\text{cr}}^{(\pm)}} = 1 \pm \omega = 1 \pm \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}}, \quad (62)$$

where the critical line $g = g_{\text{cr}}^{(+)}$ behaves as a UV fixed point, while the non-critical line $g = g_{\text{cr}}^{(-)}$ an IR fixed point:

$$\beta^{NP}(g) = \frac{\partial g}{\partial \ln \Lambda} \Big|_{\alpha, m_F} = -2 \left(g - g_{\text{cr}}^{(+)} \right) \left(g - g_{\text{cr}}^{(-)} \right). \quad (63)$$

The nonperturbative running coupling near the UV fixed point $g = g_{\text{cr}}^{(+)}$ is given as $g(\mu) = g_{\text{cr}}^{(+)} (1 + \frac{m_F^{2\omega}}{\mu^{2\omega} - m_F^{2\omega}})$ for $g > g_{\text{cr}}^{(+)} (\mu > m_F)$ and $g(\mu) = g_{\text{cr}}^{(+)} (1 - \frac{m_F^{2\omega}}{\mu^{2\omega} + m_F^{2\omega}})$ for $g < g_{\text{cr}}^{(+)}$. At $\alpha \rightarrow \alpha_{\text{cr}}$, the fusion of the UV and IR fixed points takes place: $g_{\text{cr}}^{(+)} = g_{\text{cr}}^{(-)} = 1/4$, and hence $\beta^{NP}(g) = -2(g - g_*)^2$ ($g_* = 1/4$) [12, 74]. This beta function again has a multiple zero but not a simple zero at UV=IR fixed point, with essential singularity scaling $m_F^2 = \Lambda^2 \exp(-1/(g - g_*))$, similarly to conformal phase transition at $\alpha_* = \alpha_{\text{cr}}$ in the walking gauge theory without four-fermion coupling [47, 49, 50] (See the next subsection).

The outstanding feature of the gauged NJL model with $\alpha \neq 0, \omega < 1$ is the renormalizability (in the sense of nontriviality, or no Landau pole)[12], when the gauge coupling is walking, $\alpha(\mu^2) \approx \text{const.}$, with the four-fermion interaction having the full dimension $2 < D = 2(3 - \gamma_m) = 4 - 2\omega < 4$ (relevant operator, or super renormalizable), including $D \simeq 2(1 + A/\ln \mu^2) > 2$ with a moderately walking small coupling $\omega \simeq 1 - \frac{\alpha}{2\alpha_{\text{cr}}} \simeq 1 - \gamma_m$ ($\gamma_m(\mu) \sim A/\ln \mu^2$) with $A = 18C_2/(11N_C - 2N_F) > 1$, in sharp contrast to the pure (non-gauged) NJL model which is a trivial theory having a Landau pole.

E. Conformal Phase Transition

We now discuss a salient feature of the phase transition at $\alpha = \alpha_{\text{cr}}$, what we call Conformal Phase Transition [45], which is characterized by the Miransky-BKT type non-analytic scaling. Let us first discuss the exact chiral limit $m_0 \equiv 0$.

In the conformal phase $\alpha \leq \alpha_{\text{cr}}$, there is no bound state (dubbed “unparticle” phase), although there is a UV scale Λ which is identified with the intrinsic scale Λ_{TC} generated quantum mechanically (already by the perturbation) by the regularization as manifested in a form of the (perturbative) trace anomaly. It should be emphasized that just on the critical point $\alpha = \alpha_{\text{cr}}$ the SSB does not take place in the same way as for $\alpha < \alpha_{\text{cr}}$, and hence there are no bound states at all. In fact the solution of the ladder SD equation at $\alpha = \alpha_{\text{cr}}$ takes the asymptotic form at $x \gg m_F^2$:

$$\Sigma(x) = \xi \cdot {}_2F_1(1/2, 1/2.2; -x) \sim \frac{2\xi}{\pi} \frac{m_F^2}{\sqrt{x}} \left(\ln\left(\frac{16x}{m_F^2}\right) - 2 \right) \quad (\alpha = \alpha_{\text{cr}}), \quad (64)$$

^{#13} The NJL four-fermion coupling can be treated effectively as if a strong asymptotically nonfree gauge coupling $\alpha(x) = g \frac{x}{\Lambda^2} \theta(\Lambda^2 - x)$ in the improved ladder kernel in Eq.(14).

which cannot satisfy the UV boundary condition Eq.(24) for $m_0 = 0$, thus $\Sigma(x) \equiv 0$ ^{#14}.

On the other hand, in the SSB phase $\alpha > \alpha_{\text{cr}}$, bound states do appear with the mass on the order of the SSB scale $\mathcal{O}(m_F) \ll \Lambda$ up to factors depending on N_F and N_C . Hence the bound states spectra change discontinuously across the phase transition point, although the order parameter m_F smoothly is changed as $m_F \rightarrow 0$ as $\alpha \searrow \alpha_{\text{cr}}$ to the value $m_F \equiv 0$ for $\alpha \leq \alpha_{\text{cr}}$ [45]. For $\alpha > \alpha_{\text{cr}}$ the massive bound states with masses proportional to m_F approach to zero when $\alpha \searrow \alpha_{\text{cr}}$ according to the Miransky-BKT scaling, while the NG bosons of the chiral symmetry are exactly massless, all of which (including the NG bosons) suddenly disappear when we reach the point $\alpha = \alpha_{\text{cr}}$. Hence it cannot be described by the Ginzburg-Landau effective theory (linear sigma model)[45]. This peculiarity is closely connected to the non-analytic form of the Miransky-BKT scaling in Eq.(44): The mass of the bound state A (other than the NG bosons of the chiral symmetry), M_A , has a *universal scaling function* $f\left(\frac{\alpha}{\alpha_{\text{cr}}}\right)$ in the same form as the dynamical mass of the technifermions m_F up to a constant $C_A(r)$ depending on the each bound state [28, 29]:

$$\frac{M_A}{\Lambda} \simeq C_A(r) \cdot f\left(\frac{\alpha}{\alpha_{\text{cr}}}\right), \quad f\left(\frac{\alpha}{\alpha_{\text{cr}}}\right) = \exp\left(-\frac{\pi}{\sqrt{\frac{\alpha}{\alpha_{\text{cr}}} - 1}}\right) \ll 1 \quad (\alpha > \alpha_{\text{cr}}), \quad (65)$$

where $M_A/\Lambda \ll 1$ can be tuned only for $\alpha > \alpha_{\text{cr}}$. This is an essential difference of the walking theory from the ordinary QCD, where all the light bound states (except the NG boson pions) have masses on the same order as the intrinsic scale $M_A = \mathcal{O}(m_F) = \mathcal{O}(\Lambda_{\text{QCD}})$: $M_A/\Lambda_{\text{QCD}} = \mathcal{O}(1)$ having no limit going to zero, in sharp contrast to the walking theory. In the case at hand, all the light bound states have vanishing masses towards the criticality [75] in a universal way $f\left(\frac{\alpha}{\alpha_{\text{cr}}}\right) \rightarrow 0$ as $\alpha \rightarrow \alpha_{\text{cr}}$ up to a constant $C_A(r)$ above as a consequence of the scale symmetry, and hence are a kind of “dormant NG bosons” of spontaneously broken scale (or chiral) symmetry existing only in the broken phase (without the exact massless point): $M_A/M_B \rightarrow \text{const.} \neq 0, \infty$ ($\alpha \rightarrow \alpha_{\text{cr}}$).

As we shall discuss later, the coefficient $C_A(r)$ for the spectra other than the TD may depend on $r = N_F/N_C$ particularly in the anti-Veneziano limit, since only the TD has the mass subject to the explicit breaking of the scale symmetry characterized by m_F , $M_\phi \sim m_F$, while others (except for technipions) reflect the SSB of the scale symmetry characterized by the dilaton decay constant $F_\phi \sim \sqrt{N_F N_C} m_F$: (Technipions have masses $M_\pi \gg m_F$, see Eq.(60))

$$\frac{M_\phi}{M_A} \sim \frac{1}{C_A(r)} \ll 1 \quad (r \rightarrow r_{\text{cr}}). \quad (66)$$

Thus the TD does tend to be massless (NG boson-like) faster than the others when approaching the criticality in a particular limit $N_C \rightarrow \infty$, $\lambda \equiv N_C \alpha = \text{fixed}$, $r \equiv N_F/N_C = \text{fixed}(\gg 1)$ (“anti-Veneziano limit” in distinction to the original Veneziano limit $r \ll 1$).

Note that the IR fixed point in the large N_F QCD as a model of the ladder coupling in the anti-Veneziano limit reads [76]

$$\frac{\alpha}{\alpha_{\text{cr}}} = \frac{N_C \alpha_*}{N_C \alpha_{\text{cr}}} \rightarrow \frac{66 - 12r}{13r - 34}, \quad f\left(\frac{\alpha}{\alpha_{\text{cr}}}\right) \rightarrow \tilde{f}(r) \quad (N_C \rightarrow \infty, \frac{34}{13} < r \equiv \frac{N_F}{N_C} < \frac{11}{2}), \quad (67)$$

which is an almost continuous parameter and hence $\frac{\alpha}{\alpha_{\text{cr}}}(> 1)$ can be tuned arbitrarily close to 1 by tuning the ratio $r \equiv N_F/N_C \nearrow r_{\text{cr}} = 4$ ^{#15}. Thus the conformal phase transition as a continuous (non-analytic) phase transition can also be realized in the large N_F QCD in the anti-Veneziano limit. Also note that the intrinsic scale Λ_{TC} as well as m_F scales as $\sim N_F^0, N_C^0$ (fixed) in that limit, while the coupling scales like $\alpha_* \simeq \alpha_{\text{cr}} = \mathcal{O}(1/N_C) (\rightarrow 0)$

^{#14} When $m_0 \neq 0$, this is an explicit breaking solution: $m_0 = (x\Sigma(x))'|_{x=\Lambda^2} = Z_m m_R$, with $Z_m = \frac{2\xi}{\pi} \frac{m_R}{\Lambda} \ln(\frac{4\Lambda}{m_R})$. This yields $\gamma_m(\mu) = 1 - 1/\ln(\frac{4\mu}{m_R})$ and the OPE: $\Sigma(x) \sim m_R e^{-\int^t dt' \gamma_m(t')} \sim \frac{m_R^2}{\sqrt{x}} \ln(\frac{16x}{m_R^2})$, with $t \equiv \ln(4\sqrt{x}/m_R)$, in agreement with Eq.(64) up to trivial factors. Appearance of the log factor is peculiarity of the conformal phase at $\alpha = \alpha_{\text{cr}}$ due to the collide/cancellation of the two terms, ω and $-\omega$, at $\omega = 0$ in Eq.(30). In the SSB phase satisfying Eq.(44), no such a log factor exists when $\alpha \rightarrow \alpha_{\text{cr}} + 0$ ($m_F/\Lambda \rightarrow 0$), as already noted in sub-subsection IID 2.

^{#15} The value $N_F^{\text{cr}} \simeq 4N_C = 12$ ($N_C = 3$)[44] should not be taken seriously, since it is the result of two crude approximations: The IR fixed point value α_* of the two-loop beta function having a big error from higher loops in a scheme-dependent way [76] is not reliable near the lower end of N_F/N_C where the loop expansion breaks down as its value $N_C \alpha_*$ is of $\mathcal{O}(1)$, and could trigger the spontaneous chiral symmetry breaking which washes out the IR fixed point (even though $\alpha_* \sim 1/N_C \ll 1$, since the critical coupling also behaves as $\alpha_{\text{cr}} \sim 1/C_2 \sim 1/N_C$). Also the critical value α_{cr} of the ladder SD equation is subject to 20 – 30 percent errors. Indeed lattice results suggest $N_F^{\text{cr}} \simeq 8$ for $N_C = 3$ [37].

and hence the *spontaneous symmetry breaking is triggered by the weak coupling*, although the “effective coupling” $N_C \alpha_* \simeq N_C \alpha_{\text{cr}} \simeq 2\pi/3$ is strong. Hence the ladder approximation is expected to give a better result in the anti-Veneziano limit. This is somewhat analogous to the $1/N_C$ expansion of the NJL model with the coupling $G \sim g/\Lambda^2$: Although the effective critical coupling is strong, $g_{\text{cr}}^{\text{eff}} = N_C \cdot g_{\text{cr}} = 1$, the coupling g as well as g_{cr} scales like $1/N_C$, which justifies the NJL gap equation valid at the leading order of $1/N_C$.

This is the essence of the WTC where the explicit breaking of the scale symmetry is tiny compared with the intrinsic scale Λ_{TC} : $m_F \ll \Lambda_{\text{TC}}$, which is in contrast to the ordinary QCD where $m_F \sim \Lambda_{\text{QCD}}$ with the scale symmetry violated completely. In fact these properties are the universal features of the WTC not restricted to the ladder SD equation. We in fact find that the ladder SD results are useful for describing the 125 GeV Higgs as the TD, *not merely qualitatively but also quantitatively* in spite of the crude approximation.

III. NONPERTURBATIVE TRACE ANOMALY

A. Nonperturbative Trace Anomaly and PCDC

When the chiral symmetry is spontaneously broken, $\langle \bar{F}F \rangle \neq 0$, the scale symmetry is also spontaneously broken in the vacuum with the condensate of the chiral operator $\bar{F}F$ transforming nontrivially under the scale transformation. This leads to the TD as a NG boson of the scale symmetry. The TD is actually not massless and thus is a pseudo NG boson, since the scale symmetry is *broken also explicitly by the same chiral condensate* as that breaks it spontaneously with a mass scale small compared with the intrinsic scale, $m_F \ll \Lambda = \Lambda_{\text{TC}}$. In fact such a small mass generation in Eq.(2) washes out the would-be IR fixed point $\alpha \simeq \alpha_*$ in the deep IR region $\mu < m_F$, namely breaks the scale-invariance (nonrunning behavior or the perturbative IR fixed point) of the input coupling.

As we discussed in subsection IID 1, the nonperturbative running of the coupling is induced by the generation of m_F through the regularization of the same chiral condensate as that breaks the scale symmetry spontaneously, where the intrinsic scale Λ_{TC} (already generated by the perturbative regularization as in Eqs.(18) and (20)) plays a role of regulator responsible for the *nonperturbative trace anomaly* [69] in distinction from the usual trace anomaly in the perturbation in Eq.(20):

$$\begin{aligned} \langle \partial_\mu D^\mu \rangle &= \langle (\theta_\mu^\mu) \rangle^{(NP)} \equiv \langle \theta_\mu^\mu \rangle^{(full)} - \langle \theta_\mu^\mu \rangle^{(perturbative)} = \frac{\beta^{(NP)}(\alpha)}{4\alpha} \langle G_{\mu\nu}^2 \rangle^{(NP)}, \\ \langle G_{\mu\nu}^2 \rangle^{(NP)} &\equiv \langle G_{\mu\nu}^2 \rangle^{(full)} - \langle G_{\mu\nu}^2 \rangle^{(perturbative)}. \end{aligned} \quad (68)$$

The formal proof of this relation was given [69] in terms of functional method for the Cornwall-Jackiw-Tomboulis effective potential $V[\Sigma(x)]$ whose stationary condition is the SD equation. The solution of the SD equation $\bar{\Sigma}(x)$ yields the vacuum energy $E = V[\bar{\Sigma}(x)]$ and $\langle \theta_0^0 \rangle = \langle \partial_\mu D^\mu \rangle = \langle (\theta_\mu^\mu) \rangle^{(NP)} = 4E$. The IR conformality is manifested in the fact that the relevant mass scale m_F is tiny, compared with that of the perturbative trace anomaly, $m_F \ll \Lambda_{\text{TC}}$, $\langle \theta_\mu^\mu \rangle^{(perturbative)} = -\mathcal{O}(N_F N_C \Lambda_{\text{TC}}^4)$ in Eq.(20). This is in sharp contrast to the ordinary QCD, where $m_F \simeq \Lambda_{\text{QCD}}$ and hence $\langle \theta_\mu^\mu \rangle \simeq \langle \theta_\mu^\mu \rangle^{(perturbative)}$, without the walking region and the IR conformality.

Based on this approximate scale symmetry in WTC, the light TD as a pseudo NG boson was predicted [6, 7] via the anomalous WT identity for the scale symmetry, so-called the PCDC relation (Eqs.(6),(8) and (9) of Ref. [7]):

$$\begin{aligned} M_\phi^2 F_\phi^2 &= -F_\phi \langle 0 | \partial^\mu D_\mu | \phi \rangle = -d_\theta \langle 0 | \theta_\mu^\mu | 0 \rangle^{(NP)} = -\frac{\beta^{(NP)}(\alpha)}{\alpha} \langle G_{\mu\nu}^2 \rangle^{(NP)} = \mathcal{O}(N_F N_C m_F^4) \\ &\ll -d_\theta \langle \theta_\mu^\mu \rangle^{(perturbative)} = \mathcal{O}(N_F N_C \Lambda_{\text{TC}}^4), \end{aligned} \quad (69)$$

where D_μ is the dilatation current and F_ϕ is the decay constant of ϕ defined as $\langle 0 | D_\mu | \phi(q) \rangle = -i F_\phi q_\mu$, and d_θ ($= 4$) is the dimension of $\theta_{\mu\nu}$. This is in sharp contrast to the ordinary QCD where $m_F \sim \Lambda_{\text{QCD}}$ and hence $|\langle \theta_\mu^\mu \rangle| = \mathcal{O}(m_F^4) = \mathcal{O}(\Lambda_{\text{QCD}}^4) = |\langle \theta_\mu^\mu \rangle^{(perturbative)}|$, totally lacking the scale symmetry. Note that $\alpha \sim \alpha_* \sim \alpha_{\text{cr}} \sim 1/N_C$ and $\beta^{(NP)}(\alpha) \sim 1/N_C$ in the anti-Veneziano limit $N_C \rightarrow \infty$ with $r \equiv N_F/N_C = \text{fixed} (\gg 1)$, so that we have $\frac{\beta^{(NP)}(\alpha)}{\alpha} \sim N_C^0$. (This is also the case for the perturbative beta function, see Eq.(16).)

B. RG invariance of the Nonperturbative trace anomaly

Here we show the RG invariance of the nonperturbative trace anomaly $\frac{\beta^{(NP)}(\alpha)}{4\alpha} \langle G_{\nu\lambda}^2 \rangle^{(NP)}$ in the ladder approximation:

$$\langle \partial^\mu D_\mu \rangle = \langle (\theta_\mu^\mu) \rangle^{(NP)} = \frac{\beta^{(NP)}(\alpha(\mu))}{4\alpha(\mu)} \langle G_{\nu\lambda}^2 \rangle_{(\mu)}^{(NP)}. \quad (70)$$

Based on the result of Ref.[47], we shall show that the RG invariance is realized in a nontrivial manner: The dependence of the renormalization point μ is precisely cancelled among $\beta^{(NP)}(\alpha(\mu))/(4\alpha(\mu)) \sim -(\frac{\alpha(\mu)}{\alpha_{cr}} - 1)^{3/2} \sim -1/\ln^3(\mu/m_F)$ and $\langle G_{\nu\lambda}^2 \rangle_{(\mu)}^{(NP)} \sim (\frac{\alpha(\mu)}{\alpha_{cr}} - 1)^{-3/2} \sim \ln^3(\mu/m_F)$ for $m_F < \mu < \Lambda_{TC}$, thereby yielding the same result as that of the vacuum energy calculation in Ref. [51]. Comparing Eq.(70) with Eq.(20), we see that the resultant trace anomaly of order $\mathcal{O}(m_F^4)$ is much smaller than the trace anomaly of order $\mathcal{O}(\Lambda_{TC}^4)$ related to the fundamental scale Λ_{TC} of the theory, and hence the use of the PCDC Eq. (69) is justified.

Let us first calculate the nonperturbative gluon condensate induced not from the gluon loop (already subtracted out) but from the fermion loop with the technifermion having dynamical mass m_F , which is calculated at the two-loop level with the technifermion propagator as given in the ladder SD equation [69]:

$$\langle G_{\mu\nu}^2 \rangle^{(NP)} = -\frac{2ig^2 N_F N_C}{(2\pi)^8} \int d^4k d^4p \text{tr}[S_F(p)\gamma_\mu S_F(k)\gamma_\nu] D^{\mu\nu}(p-k). \quad (71)$$

By using the ladder SD equation for $S_F(p)$ in Eq.(13) with the nonrunning coupling, Eq.(71) can be rewritten into a simpler form

$$\begin{aligned} \langle G_{\mu\nu}^2 \rangle^{(NP)} &= -\frac{2ig^2 N_F N_C}{(2\pi)^4} \int d^4p \text{tr}[1 - S_F(p)S^{-1}(p)] \\ &= \frac{N_F N_C}{2\pi^2} \int_0^{\Lambda^2} dx \frac{x\Sigma^2(x)}{x + \Sigma^2(x)} = \frac{N_F N_C}{2\pi^2} \int^{\Lambda^2} dx \left[\Sigma^2(x) - \frac{\Sigma^4(x)}{x + \Sigma^2(x)} \right], \end{aligned} \quad (72)$$

where the second term of the integral yields correction of order $\mathcal{O}(m_F^8/\Lambda^4)$, since $\Sigma(x) \sim m_F^2/\sqrt{x}$, and hence can be ignored. For the high energy region where $x \gg m_F^2$, the mass function $\Sigma(x)$ takes the form given by Eq.(42). Using $\Sigma(x)$ in Eq.(42), we find $\int^{\Lambda^2} dx \Sigma^2(x) \simeq \xi^2 m_F^4 \frac{8\text{cth}\frac{\pi\tilde{\omega}}{2}}{\pi\tilde{\omega}(\tilde{\omega}^2+1)} \ln\left(\frac{4\Lambda}{m_F}\right) = \frac{16\xi^2}{\pi^2\tilde{\omega}^2} m_F^4 \ln\left(\frac{4\Lambda}{m_F}\right) [1 + \mathcal{O}(\tilde{\omega}^2)]$. From Eq.(72), we find

$$\begin{aligned} \langle G_{\mu\nu}^2 \rangle^{(NP)} &\simeq \frac{N_F N_C}{2\pi^2} \cdot m_F^4 \frac{16\xi^2}{\pi^2\tilde{\omega}^2} \ln\left(\frac{4\Lambda}{m_F}\right) \simeq N_F N_C \frac{8\xi^2}{\pi^3} m_F^4 \cdot \left(\frac{1}{\pi} \ln\left(\frac{4\Lambda}{m_F}\right)\right)^3 \\ &\simeq N_F N_C \frac{8\xi^2}{\pi^3} m_F^4 \cdot \left(\frac{\alpha}{\alpha_{cr}} - 1\right)^{-3/2}, \end{aligned} \quad (73)$$

up to factor of $(1 + \mathcal{O}(\tilde{\omega}^2))$, where we have used the Miransky scaling Eq.(44): $\tilde{\omega} = \sqrt{\alpha/\alpha_{cr} - 1} = \pi/\ln(4\Lambda/m_F)$. Thus the gluon condensate is diverging as $\left(\ln\frac{\Lambda}{m_F}\right)^3$ much faster than the QCD-like theory with divergence $\ln\frac{\Lambda}{m_F}$ as noted in Ref.[47].

Note that the divergence of $\langle G_{\mu\nu}^2 \rangle^{(NP)}$ is of the same origin as that for the amplification of the symmetry violation such as the technipion mass coming from the UV contributions enhanced by the large anomalous dimension: $\Sigma(x) \sim \frac{m_F^3}{x} (\frac{x}{m_F^2})^{\gamma_m/2} \sim m_F^2/\sqrt{x}$. Do not confuse it from the log divergence of the gluon condensate in the ordinary QCD, which comes from the gluon loop in contrast to the present case coming from the fermion loop. We shall discuss it later.

Actually, it was found [47] that the divergence $\sim \left(\ln\left(\frac{4\Lambda}{m_F}\right)\right)^3$ in Eq.(73) is precisely cancelled by the vanishing factor of the nonperturbative beta function in Eq.(45):

$$\beta^{(NP)}(\alpha) = -\frac{2\alpha_{cr}}{\pi} \left(\frac{1}{\pi} \ln\left(\frac{4\Lambda}{m_F}\right)\right)^{-3} = -\frac{2\alpha}{\pi} \left(\frac{\alpha}{\alpha_{cr}} - 1\right)^{3/2} [1 + \tilde{\omega}^2]^{-1}, \quad (74)$$

such that the trace anomaly of the energy momentum tensor $\langle \theta_\mu^\mu \rangle$ is given by

$$\langle \theta_\mu^\mu \rangle^{(NP)} = \frac{\beta^{(NP)}(\alpha)}{4\alpha} \langle G_{\mu\nu}^2 \rangle^{(NP)} \simeq -N_F N_C \frac{4\xi^2}{\pi^4} m_F^4 [1 + \mathcal{O}(\tilde{\omega}^2)]. \quad (75)$$

Thus the smallness of $\beta^{(NP)}(\alpha)$ as manifestation of the approximate scale symmetry is in fact operative by canceling the otherwise amplified symmetry violation effects of the large anomalous dimension, and hence keeping the nonperturbative trace anomaly, the explicit breaking of the scale symmetry, on the order of m_F^4 .

On the other hand, the direct computation of $\langle\theta_\mu^\mu\rangle$ through the vacuum energy $\langle\theta_\mu^\mu\rangle = 4\langle\theta_0^0\rangle$ is [51]:

$$\begin{aligned}\langle\theta_\mu^\mu\rangle^{(NP)} &= 4V[\Sigma(x)] = -\frac{N_F N_C}{4\pi^2} \left[\Lambda^4 \ln \left(1 + \frac{\Sigma(\Lambda^2)}{\Lambda^2} \right) \right] \simeq -\frac{N_F N_C}{4\pi^2} \Lambda^2 \Sigma^2(\Lambda^2) \\ &\simeq -\frac{N_F N_C}{4\pi^2} \xi^2 m_F^4 \frac{8 \operatorname{cth} \frac{\pi \tilde{\omega}}{2}}{\pi \tilde{\omega} (\tilde{\omega}^2 + 1)} \sin \tilde{\omega}^2 = -N_F N_C \frac{4\xi^2}{\pi^4} m_F^4 [1 + \mathcal{O}(\tilde{\omega}^2)].\end{aligned}\quad (76)$$

Let us take $\Lambda \rightarrow \infty$ such that $\alpha(\Lambda) \rightarrow \alpha_{\text{cr}}$ ($\tilde{\omega} \rightarrow 0$), then Eq.(75) precisely coincides with Eq.(76). Thus the three independent calculations of different quantities are consistent with each other within the ladder approximation ^{#16}.

This is reformulated in terms of the nonperturbative running $\alpha(\mu)$ in the renormalization defined in Eq.(45) as

$$\langle\theta_\mu^\mu\rangle^{(NP)} = \frac{\beta^{(NP)}(\alpha(\mu))}{4\alpha(\mu)} \langle G_{\nu\lambda}^2 \rangle_{(\mu)}^{(NP)} = -N_F N_C \frac{4\xi^2}{\pi^4} m_F^4. \quad (77)$$

Then the nonperturbative trace anomaly $\langle\theta_\mu^\mu\rangle$ is written in the manifestly RG-independent way in the ladder approximation as it should be.

Such an RG invariance by cancellation is a well-known fact for the perturbative trace anomaly but is explicitly recognized for the first time for the nonperturbative trace anomaly. It is in fact well-known that the perturbative trace anomaly is RG invariant, i.e., independent of the renormalization point μ . In the chiral limit it reads $\langle\theta_\mu^\mu\rangle = \beta(\alpha)/(4\alpha) \langle G_{\mu\nu}^2 \rangle$ which is RG invariant in such a way that $\beta(\alpha)/(4\alpha) \sim \alpha \sim 1/\ln(\mu/\Lambda_{\text{QCD}})$ precisely cancels the divergence in $\langle G_{\mu\nu}^2 \rangle \sim \ln(\mu/\Lambda_{\text{QCD}})$ as $\mu \rightarrow \infty$. This is also applied to the WTC in the UV region $\mu > \Lambda_{\text{TC}}$, where the perturbative trace anomaly in Eq.(20) is obviously RG invariant in the same way as in the ordinary QCD. Note that in the usual QCD the scale-invariance appear to exist “formally” in the UV region $\mu \gg \Lambda_{\text{QCD}}$ due to the vanishing $\beta(\alpha)/(4\alpha) \sim 1/\ln(\mu/\Lambda_{\text{QCD}}) \rightarrow 0$ at the trivial UV fixed point $\alpha_* = 0$, which is however compensated by the diverging gluon condensate $\langle G_{\mu\nu}^2 \rangle \sim \ln(\mu/\Lambda_{\text{QCD}})$, and hence the scale invariance in QCD exists nowhere.

C. Inclusion of small bare mass of technifermions: Basis for the dilaton chiral perturbation theory

For completeness we here show that with inclusion of the small bare mass m_0 or the renormalized mass (“current mass” $m_R (\ll m_F)$) of the technifermion, the ladder calculations of various quantities yield a consistent trace anomaly for the anomalous WT identity, which is the basis of the sChPT [27]. It is vital for the lattice calculations of the flavor-singlet scalar bound state as a candidate for the technidilaton, whose observed mass and decay constant should be extrapolated to the chiral limit.

Here we explicitly check that the ladder approximation is consistent with the anomalous WT identity for the SSB of the approximate scale symmetry (for $\gamma_m = 1$):

$$\langle\theta_\mu^\mu\rangle = \frac{\beta^{(NP)}(\alpha)}{4\alpha} \langle G_{\mu\nu}^2 \rangle + (1 + \gamma_m) N_F m_R \langle \bar{F} F \rangle_R = \frac{\beta^{(NP)}(\alpha)}{4\alpha} \langle G_{\mu\nu}^2 \rangle + 2N_F m_R \langle \bar{F} F \rangle_R. \quad (78)$$

The formal proof of this relation was also given in Ref. [69] in terms of functional method for the Cornwall-Jackiw-Tomboulis effective potential $V[\Sigma(x)]$. The relation is the basis of the sChPT [27] for the TD mass in the presence of the technifermion explicit mass (current mass) m_R . The current mass m_R and the associated-renormalized chiral condensate $\langle \bar{F} F \rangle_R$ are related to the bare mass m_0 in Eq.(24) and the bare-chiral condensate $\langle \bar{F} F \rangle_0$ involving the renormalization constant Z_m in Eq.(54) as

$$m_R = Z_m^{-1} m_0. \quad (79)$$

$$\langle \bar{F} F \rangle_R = Z_m \langle \bar{F} F \rangle_0. \quad (80)$$

^{#16} For idealized large N_C in the anti-Veneziano limit, ladder calculation becomes more reliable, as we demonstrated in Fig.2. The result of Eq.(76) based on the ladder thus becomes more reliable in the anti-Veneziano limit. As in the case of usual large N_c arguments in QCD ($N_c = 3 \rightarrow \infty$), the quantitative check of the validity of the anti-Veneziano limit for the realistic value of N_F and N_C is of course subject to the fully nonperturbative check by the lattice studies.

We then see that Eq.(78) is nothing but the chiral expansion of $\langle\theta_\mu^\mu\rangle$ and/or the dilaton mass m_ϕ (sChPT [27]):

$$\begin{aligned}\langle\theta_\mu^\mu\rangle &= \left.\langle\theta_\mu^\mu\rangle\right|_{m_R=0} + \left.\frac{\partial\langle\theta_\mu^\mu\rangle}{\partial m_0}\right|_{m_R=0} \cdot m_R, \\ \left.\langle\theta_\mu^\mu\rangle\right|_{m_R=0} &= \frac{\beta_{\text{NP}}(\alpha)}{4\alpha} \langle G_{\mu\nu}^2 \rangle \Big|_{m_R=0} = -\frac{4\xi^2}{\pi^4} N_F N_C m_F^4, \\ \left.\frac{\partial\langle\theta_\mu^\mu\rangle}{\partial m_0}\right|_{m_0=0} \cdot m_R &= 2N_F m_R \langle \bar{F}F \rangle_R,\end{aligned}\tag{81}$$

where $\left.\langle\theta_\mu^\mu\rangle\right|_{m_R=0} = -\frac{4\xi^2}{\pi^4} N_F N_C m_F^4$ is given by Eq.(75).

We shall check Eq.(81) by evaluating both sides with use of the ladder results. The bare-chiral condensate for $m_R = 0$ is given as [69]

$$\langle \bar{F}F \rangle_0 \simeq -\frac{4N_C}{\pi^3} \xi m_F^2 \Lambda,\tag{82}$$

which is converted into the renormalized condensate through the scaling relation in Eq.(80) with the $Z_m = \frac{2\xi}{\pi} \frac{m_F}{\Lambda}$ in Eq.(54):

$$\langle \bar{F}F \rangle_R = Z_m \cdot \langle \bar{F}F \rangle_0 \simeq -\frac{8N_C}{\pi^4} \xi^2 m_F^3.\tag{83}$$

On the other hand, the $\langle\theta_\mu^\mu\rangle$, with the small current mass $m_R (\ll m_F)$ included into the full mass of the technifermion $m_P \simeq m_F + m_R$, is given as [69]

$$\langle\theta_\mu^\mu\rangle \simeq -\xi^2 \frac{4N_F N_C}{\pi^4} (m_R + m_F)^4 \simeq \frac{\beta_{\text{NP}}(\alpha)}{4\alpha} \langle G_{\mu\nu}^2 \rangle \Big|_{m_R=0} + \left.\frac{\partial\langle\theta_\mu^\mu\rangle}{\partial m_R}\right|_{m_R=0} m_R,\tag{84}$$

with

$$\left.\frac{\partial\langle\theta_\mu^\mu\rangle}{\partial m_R}\right|_{m_R=0} \simeq -\frac{16N_F N_C}{\pi^4} \xi^2 m_F^3 = 2N_F \langle \bar{F}F \rangle_R,\tag{85}$$

which reproduces Eq.(78).

It is straightforward to write down the effective theory to realize the relation, in Eq.(78) namely the sChPT [27]:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{(2)}^{\text{inv}} + \mathcal{L}_{(2)\text{anomaly}}^S + \mathcal{L}_{(2)\text{mass}}^S + \mathcal{L}_{(4)}, \\ \mathcal{L}_{(2)}^{\text{inv}} &= \frac{F_\phi^2}{2} (\partial_\mu \chi)^2 + \frac{F_\pi^2}{4} \chi^2 \text{tr}[\partial_\mu U^\dagger \partial^\mu U], \\ \mathcal{L}_{(2)\text{anomaly}}^S &= -\frac{F_\phi^2}{4} m_\phi^2 \chi^4 \left(\log \frac{\chi}{S} - \frac{1}{4} \right), \\ \mathcal{L}_{(2)\text{mass}}^S &= \frac{F_\pi^2}{4} \left(\frac{\chi}{S} \right)^{3-\gamma_m} \cdot S^4 \text{tr}[\mathcal{M}^\dagger U + U^\dagger \mathcal{M}] - \frac{(3-\gamma_m)F_\pi^2}{8} \chi^4 \cdot (N_F \text{tr}[(\mathcal{M}^\dagger \mathcal{M})])^{1/2},\end{aligned}\tag{86}$$

where $U(x) = e^{2i\pi(x)/F_\pi}$, $\chi(x) = e^{\phi(x)/F_\phi}$ ($\langle\chi\rangle = 1, \langle\phi\rangle = 0$) are nonlinear bases for the chiral and scale transformations, and \mathcal{M} and $S(x)$ are chiral and scale spurion fields transforming in the same way as $U(x)$ and $\chi(x)$, respectively, with $\langle\mathcal{M}\rangle = m_R$, $\langle S(x)\rangle = 1$. $\mathcal{L}_{(4)}$ contains the $\mathcal{O}(p^4)$ counter terms of the ChPT with $M_\phi^2 = \mathcal{O}(p^2)$ and explicitly given in Ref. [27]. The $\mathcal{O}(p^2)$ terms in Eq.(86) lead to the TD mass formula [27]:

$$M_\phi^2 = m_\phi^2 + \frac{(3-\gamma_m)(1+\gamma_m)}{4} \cdot \frac{2N_F F_\pi^2}{F_\phi^2} m_\pi^2 \simeq m_\phi^2 + \frac{2N_F F_\pi^2}{F_\phi^2} m_\pi^2,\tag{87}$$

where $m_\phi = M_\phi|_{m_R=0}$ is the TD mass in the chiral limit. The same result is also derived directly from the anomalous WT identity for the scale symmetry and chiral WT identity. The result is useful for determining the mass and decay constant of TD by the lattice simulations through chiral extrapolation.

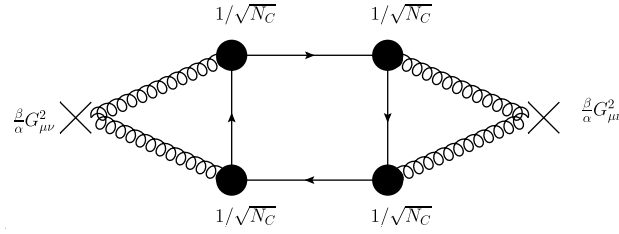


FIG. 4: The Feynman graph and the large N_C and N_F countings for the fermion loop contribution to the correlation function of gluon condensate operators reflecting the anti-Veneziano limit.

Note that the nonperturbative trace anomaly is given by $\langle \theta_\mu^\mu \rangle = \langle \partial^\mu D_\mu \rangle = \langle \delta_D \mathcal{L}_{(2)\text{anomaly}}^S \rangle = -m_\phi^2 F_\phi^2 \langle \chi \rangle^4 / 4 = -m_\phi^2 F_\phi^2 / 4$ for $S(x) = 1$, in accord with the PCDC relation, Eq.(69), where $\delta_D \chi = \chi + x^\mu \partial_\mu \chi$ is the dilatation transformation. The form of $\mathcal{L}_{(2)\text{anomaly}}^S$ is unique in the sense that it correctly reproduces the nonperturbative trace anomaly through the log factor and the factor $-1/4$ in the parenthesis is crucial both for eliminating the ϕ tad pole (linear term in ϕ) so as to have the correct vacuum $\langle \chi \rangle = 1$ ($\langle \phi \rangle = 0$) and the correct vacuum energy $E = \langle \theta_0^0 \rangle = \langle \theta_\mu^\mu \rangle / 4 = -m_\phi^2 F_\phi^2 / 16$, as well as the correct mass term of ϕ , see later Eq.(104). The form has a characteristic log form of reflecting the trace anomaly generated by the nonperturbative dynamics, similarly to the Coleman-Weinberg potential which is generated by the perturbative trace anomaly.

IV. MASS AND DECAY CONSTANT OF THE TECHNIDILATON

From Eq.(77) the PCDC relation in the ladder approximation reads

$$M_\phi^2 F_\phi^2 = -F_\phi \langle 0 | \partial_\mu D^\mu | \phi \rangle = -4 \langle 0 | \theta_\mu^\mu | 0 \rangle^{(NP)} = -\frac{\beta^{(NP)}(\alpha(\mu))}{\alpha(\mu)} \langle G_{\mu\nu}^2 \rangle_{(\mu)}^{(NP)} = N_F N_C \left(\frac{16\xi^2}{\pi^4} m_F^4 \right). \quad (88)$$

Let us consider the saturation of the anomalous WT identity for the scale symmetry in the anti-Veneziano limit:

$$F_\phi^2 M_\phi^2 = \mathcal{F} \cdot \mathcal{T} \cdot \langle T(\partial^\mu D_\mu(x) \cdot \partial^\mu D_\mu(0)) \rangle = \mathcal{F} \cdot \mathcal{T} \cdot \left\langle T \left(\frac{\beta^{(NP)}(\alpha)}{4\alpha} G_{\mu\nu}^2(x)^{(NP)} \cdot \frac{\beta^{(NP)}(\alpha)}{4\alpha} G_{\mu\nu}^2(0)^{(NP)} \right) \right\rangle, \quad (89)$$

which is dominated by the fermion loop in Fig. 4 and hence scales like $N_F N_C^3 \alpha^2 \sim N_F N_C$, in accord with the explicit ladder computation in Eq.(88).

Instead of the notion of the nonperturbative running coupling, Eq.(4), one may use the ladder SD solution $\Sigma(x)$ in Eq.(42) and the Miransky scaling Eq.(65), in terms of the parameter N_F , with $\Lambda = \Lambda_{\text{TC}}$ fixed ($\gg m_F$), through the CBZ IR fixed point $\alpha_* (\gtrsim \alpha \gtrsim \alpha_{\text{cr}})$: [44]

$$m_F = 4\Lambda_{\text{TC}} \exp \left(-\frac{\pi}{\sqrt{\frac{\alpha_*}{\alpha_{\text{cr}}} - 1}} \right), \quad 0 < \frac{\alpha}{\alpha_{\text{cr}}} - 1 \lesssim \frac{\alpha_*}{\alpha_{\text{cr}}} - 1 = \frac{\pi^2}{\ln^2 \left(\frac{4\Lambda_{\text{TC}}}{m_F} \right)} \propto N_F^{\text{cr}} - N_F \ll 1. \quad (90)$$

Then Eq.(73) and Eq.(76) read for $\alpha_* \gtrsim \alpha \gtrsim \alpha_{\text{cr}}$ #17:

$$\langle G_{\mu\nu}^2 \rangle^{(NP)} \sim N_C N_F m_F^4 \cdot (N_F^{\text{cr}} - N_F)^{-3/2}, \quad \langle \theta_\mu^\mu \rangle^{(NP)} = -N_F N_C \frac{4\xi^2}{\pi^4} m_F^4, \quad (91)$$

from which for consistency with the trace anomaly we necessarily have the nonperturbative beta function in the broken phase $N_F < N_F^{\text{cr}}$:

$$\beta^{(NP)}(\alpha) \sim -(N_F^{\text{cr}} - N_F)^{3/2} \quad (< 0 \quad \text{for} \quad N_F < N_F^{\text{cr}}). \quad (92)$$

#17 More precisely, $\frac{\alpha_*}{\alpha_{\text{cr}}} - 1 \propto r_{\text{cr}} - r$ instead of $\propto N_F^{\text{cr}} - N_F$ in the anti-Veneziano limit, where $r = N_F / N_C$. The two-loop CBZ value of α_* plus ladder SD value of α_{cr} implies $r_{\text{cr}} = 4$ and $\frac{\alpha_*}{\alpha_{\text{cr}}} - 1 \simeq (25/18)(4 - r)$ in that limit.

This agrees with $\beta^{(NP)}(\alpha) = -\frac{\partial\alpha}{\partial\ln m_F}$ along with Eq.(90), in contrast to the arguments based on the two-loop beta function Eq.(16), $\beta^{(2-loop)}(\alpha) \sim -(N_F^{cr} - N_F)$ [77, 78], which cannot cancel the divergence of $\ln^3(\Lambda_{TC}/m_F) \sim (N_F^{cr} - N_F)^{-3/2}$. Hence our results arrive at the same for the TD mass as Eq.(88):

$$\beta^{(NP)}(\alpha) \sim -(N_F^{cr} - N_F)^{3/2}, \quad \langle G_{\mu\nu}^2 \rangle^{(NP)} \sim N_C N_F m_F^4 \cdot (N_F^{cr} - N_F)^{-3/2}, \quad (93)$$

$$\text{s.t.} \quad M_\phi^2 F_\phi^2 \simeq \frac{\beta^{(NP)}(\alpha)}{\alpha} \langle G_{\mu\nu}^2 \rangle^{(NP)} = N_F N_C \left(\frac{16\xi^2}{\pi^4} m_F^4 \right). \quad (94)$$

Note that the two-loop beta function Eq.(16) having the linear zero at the CBZ IR fixed point α_* , $\beta^{(2-loop)}(\alpha) \sim -(N_F^{cr} - N_F)$, is obviously invalid in the broken phase $\alpha_* > \alpha_{cr}$ ($N_F < N_F^{cr}$), where tuning $m_F/\Lambda_{TC} \ll 1$ should be made through the Miransky scaling Eq.(90) as $\alpha_*/\alpha_{cr} \searrow 1$ ($N_F \nearrow N_F^{cr}$). See the discussions below Eq.(45).

Thus as note in Ref. [47], the suppression effect by the small beta function $\beta^{(NP)}(\alpha)/(4\alpha) \ll 1$ as naively expected [7, 77, 78] for the $M_\phi^2 F_\phi^2$ is actually compensated by the enhancement of $\langle G_{\mu\nu}^2 \rangle^{(NP)}$ due to the large anomalous dimension $\gamma_m = 1$, both being the two sides of the same coin, characteristic to the approximate scale invariance for $\alpha \simeq \alpha_* \approx \alpha_{cr}$ ($m_F \ll \mu < \Lambda_{TC}$). Actually, it is in a more sophisticated way that the smallness of the beta function or the approximate scale symmetry is responsible for the lightness of the TD : Lightness of the TD is guaranteed first by the hierarchy $m_F \ll \Lambda_{TC}$ corresponding to the smallness of $\beta(\alpha)$, which implies the nonperturbative trace anomaly of order of $\mathcal{O}(m_F^4)$ is much smaller than the perturbative trace anomaly on the order of $\mathcal{O}(\Lambda_{TC}^4)$ [18]. Additional small hierarchy $M_\phi \ll F_\phi$ ($M_\phi \ll v_{EW}$) comes from the N_C, N_F scaling related with the same requirement $m_F \ll \Lambda_{TC}$ via more concrete setting of the anti-Veneziano limit $N_C \rightarrow \infty$ with tuning of $r \equiv N_F/N_C (\gg 1)$.

We now discuss the TD mass based on the PCDC relation, Eq.(88), in the ladder approximation. From Eq.(77) with $m_F \ll \Lambda_{TC}$, we in fact have a small nonperturbative explicit breaking of the scale symmetry: $|\langle \theta_\nu \rangle^{(NP)}| \ll |\langle \theta_\nu \rangle^{(perturbative)}|$, and hence a small TD mass compared with the intrinsic scale Λ_{TC} . Such a small pseudo NG boson mass can be estimated by the anomalous WT identity for the PCDC [7] as in Eq.(88).

As already noted in the Introduction, m_F is independent of N_F, N_C , since it is related to the N_F, N_C -independent quantity $\Lambda = \Lambda_{TC}$ via the Miransky scaling in Eq.(65), which is N_F, N_C -independent, with α/α_{cr} and/or α_*/α_{cr} is independent of N_F, N_C . Since the dilatation current $D_\mu(x)$ is sum of contributions from $N_F N_C$ fermions, and $|\phi\rangle$ is a flavor/color singlet state normalized as $1/\sqrt{N_F N_C}$, it follows that $F_\phi = \mathcal{O}(\sqrt{N_F N_C} m_F)$ by definition of F_ϕ , $\langle 0|D_\mu|\phi\rangle = -iq_\mu F_\phi$.^{#18} Hence Eq.(88) generically implies that M_ϕ is independently of N_F and N_C . From the above rough estimate $F_\phi^2 = \mathcal{O}(N_F N_C m_F^2)$, Eq.(88) reads

$$M_\phi = \mathcal{O}\left(\frac{4}{\pi^2} m_F\right) = \mathcal{O}\left(\frac{m_F}{2}\right). \quad (95)$$

Furthermore the Pagels-Stokar formula for $F_\pi^2 \simeq (N_C \xi^2 / 2\pi^2) m_F^2$ in the ladder approximation (see Eq.(B4)),

$$v_{EW}^2 = (246 \text{ GeV})^2 = N_D F_\pi^2 \simeq N_F N_C \frac{\xi^2}{4\pi^2} m_F^2 \simeq m_F^2 \left[\frac{N_F}{8} \frac{N_C}{4} \right], \quad (96)$$

with $N_D (= N_F/2)$ being the number of the electroweak doublets, which combined with Eq.(88), leads to

$$F_\phi = \mathcal{O}\left(\frac{2\pi}{\xi} v_{EW}\right) = \mathcal{O}(5 v_{EW}). \quad (97)$$

Note that both Eqs.(95) and (97) are *independently of N_F and N_C* , as far as the PCDC makes sense (as in WTC in the anti-Veneziano limit).

^{#18} This can also be seen explicitly in the linear sigma model where TD can be a radial mode ϕ (Higgs in the SM) under certain condition [60]. In the polar decomposition of the chiral field $M = HU$, where $M \sim \bar{F}_R F_L$ is a $N_F \times N_F$ complex matrix transforming as $M \rightarrow g_L M g_R^\dagger$ with $g_{L,R} \in SU(N_F)_L \times SU(N_F)_R$, and $H_{\alpha\beta} = (\phi + F_\pi)\delta_{\alpha\beta}$ and U are hermitian and unitary matrix, respectively. The decay constant of ϕ in the linear sigma model (SM) is given by $F_\phi^2 = (3 - \gamma_m)^2 \frac{N_F}{2} F_\pi^2$ [69], where γ_m is the anomalous dimension of the field M ($\gamma_m = 1$ for the case that M is a composite field $\bar{F}_R F_L$ in the WTC). Under the condition that the linear sigma model is regarded as an effective theory of the WTC [60], this would yield $F_\phi^2 \simeq N_F N_C \frac{2\xi^2}{\pi^2} m_F^2 = \mathcal{O}(N_F N_C m_F^2)$, when combined with the Pagels-Stokar formula in the ladder. (When the nonlinear sigma model limit is taken, the relation of $F_\phi^2 / [(3 - \gamma_m) F_\pi]^2 = \frac{N_F}{2}$ would become arbitrary, in agreement with our PCDC relation for TD.) In passing, the linear sigma model result coincides with the holographic estimate of the F_ϕ [23].

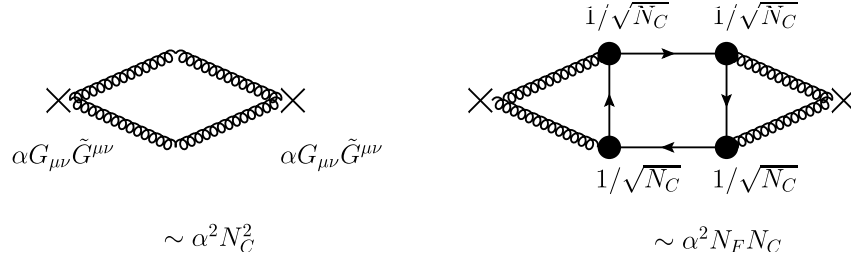


FIG. 5: The loop diagrams contributing to the correlation function of $\alpha G_{\mu\nu} \tilde{G}^{\mu\nu}$ coming from the gluon loop (left panel) and fermion loop (right panel). The large N_C and N_F scalings have also been specified.

At this point, we should comment on a widely spread folklore claiming that the natural scale of the technicolor would be $\mathcal{O}(\text{TeV})$ and hence the Higgs mass 125 GeV cannot be obtained without fine tuning. This is totally unjustified statement tinted by the naive scale up of the QCD with $N_F = 2$, $N_C = 3$ where $m_F = \mathcal{O}(650 \text{ GeV})$ from Eq.(96), in sharp contrast to $m_F \simeq 246 \text{ GeV}$ in our walking theory with $N_F = 8$, $N_C = 4$ based on the same PS formula. Moreover, the folklore presumes the naive non-relativistic estimate $M_\phi \sim 2m_F$ which would give $M_\phi = \mathcal{O}(\text{TeV})$ for $N_F = 2$ $N_C = 3$ in the QCD scale-up, where the PCDC does not make sense and no particular constraint on the flavor-singlet scalar bound state (no longer a dilaton-like object), since the ordinary QCD has no scale symmetry at all. In contrast, the approximate scale symmetry in the walking theory dictates the PCDC relation, which yields $M_\phi \simeq 125 \text{ GeV} \ll 2m_F$, instead of the above naive non-relativistic guess.

The result in fact reflects a generic scaling law,

$$\frac{M_\phi}{v_{\text{EW}}} \sim \frac{M_\phi}{F_\phi} \sim \frac{1}{\sqrt{N_C N_F}} \rightarrow 0, \quad (98)$$

independently of the ladder approximation, since it is a direct consequence of the the anti-Veneziano limit, N_F, N_C scaling of the PCDC relation $M_\phi^2 F_\phi^2 \propto N_F N_C m_F^4$ and of $F_\phi^2 \propto v_{\text{EW}}^2 \propto N_F N_C m_F^2$ coming from the definition of F_ϕ and v_{EW} in terms of the dynamical mass of the technifermions. Then in the “anti-Veneziano limit” $N_C \rightarrow \infty$ with $N_F/N_C = \text{fixed}$ ($\gg 1$, in accord with the IR conformality near the conformal window), the TD parametrically has a vanishing mass compared with the spontaneous scale-symmetry breaking scale F_ϕ ($\ll \Lambda_{\text{TC}}$): M_ϕ/F_ϕ ($\gg M_\phi/\Lambda_{\text{TC}}$) $\rightarrow 0$ [30, 32].

Thus the light TD with the mass of 125 GeV can be regarded as a pseudo NG boson in the anti-Veneziano limit near the conformal window [32]: Such a light TD is in fact similar to the η' meson in the sense that η' is widely accepted to be a pseudo-NG boson having a parametrically vanishing mass $M_{\eta'}/F_\pi = \mathcal{O}(\sqrt{N_F}/N_C) < M_{\eta'}/\Lambda_{\text{QCD}} = \mathcal{O}(\sqrt{N_F/N_C}) \rightarrow 0$ in the large N_C limit with N_F/N_C fixed ($\ll 1$) in the ordinary QCD (original Veneziano limit), a la Witten-Veneziano. In fact the anomalous chiral WT identity for $A_\mu^0(x) = \sum_{i=1}^{N_F} \bar{q}_i(x) \gamma_\mu \gamma_5 q_i(x)$ reads:

$$\begin{aligned} N_F F_\pi^2 M_{\eta'}^2 &= \mathcal{F.T.} \langle T (\partial^\mu A_\mu^0(x) \cdot \partial^\mu A_\mu^0(0)) \rangle = \mathcal{F.T.} \left\langle T \left(N_F \frac{\alpha}{4\pi} G^{\mu\nu} \tilde{G}_{\mu\nu}(x) \cdot N_F \frac{\alpha}{4\pi} G^{\mu\nu} \tilde{G}_{\mu\nu}(0) \right) \right\rangle \\ &\sim N_F^2 \alpha^2 \times [N_C^2 (\text{gluon loop, Fig.5}) + N_C^3 N_F \alpha^2 (\text{fermion loop, Fig.5})]. \end{aligned} \quad (99)$$

In the Veneziano limit $N_F/N_C \ll 1$ the gluon loop dominates the fermion loop, and hence we have

$$M_{\eta'}^2 \sim \frac{N_F}{F_\pi^2} \Lambda_{\text{QCD}}^4 \sim \frac{N_F}{N_C} \Lambda_{\text{QCD}}^2 \ll \Lambda_{\text{QCD}}^2 \quad \frac{M_{\eta'}^2}{F_\pi^2} \sim \frac{N_F}{N_C^2} \ll 1. \quad (100)$$

Thus the TD in the anti-Veneziano limit and η' in the Veneziano limit are resemblant.

What about the η' in the anti-Veneziano limit, then? (No TD exists in the Veneziano limit, since it is not a walking theory.) From Eq.(89) and Fig. 5, we see the fermion loop dominates the gluon loop, contrary to the Veneziano limit. Then we infer

$$M_{\eta'}^2 \sim \frac{N_F \alpha^2}{F_\pi^2} (N_C^3 N_F \alpha^2 m_F^2) \sim \left(\frac{N_F}{N_C} m_F \right)^2 \gg m_F^2, \quad (101)$$

where we have again subtracted the perturbative contribution to the $U(1)_A$ anomaly. This could be tested on the lattice simulation [79]. In the anti-Veneziano limit the η' mass does not go to zero and hence has no NG boson

nature in contrast to the TD. In the walking case with $N_C/N_F \gg 1$ and $m_F \ll \Lambda_{\text{TC}}$, a simple scaling suggests that $M_\phi^2 = \mathcal{O}(m_F^2)$ and $M_{\eta'}^2 = \mathcal{O}(N_F^2/N_C^2) m_F^2 (\gg M_\phi^2)$.

For the phenomenological studies, the PCDC in Eq.(88) together with the Pagels-Stokar formula in Eq.(96) yields a more concrete result:

$$M_\phi^2 \simeq \left(\frac{v_{\text{EW}}}{2}\right)^2 \cdot \left(\frac{5 v_{\text{EW}}}{F_\phi}\right)^2 \cdot \left[\frac{8}{N_F} \frac{4}{N_C}\right]. \quad (102)$$

which is in accord with [47] based on the improved ladder result (with the two-loop coupling as the input coupling). It was first pointed out in Ref. [20] that this ladder PCDC result accommodates the 125 GeV Higgs with $F_\phi = \mathcal{O}(\text{TeV})$ for the one-family model with $N_F = 8$.

Phenomenologically, the most interesting case is the one-family model ($N_F = 8$) [17, 40] with $N_C = 4$, where we have $m_F \simeq v_{\text{EW}} = 246 \text{ GeV}$ (Eq.(96)), and Eq.(102) quite naturally accommodates the realistic point [21, 22]:

$$M_\phi \simeq \frac{v_{\text{EW}}}{2} \simeq \frac{m_F}{2} \simeq 125 \text{ GeV}, \quad F_\phi \simeq 5 v_{\text{EW}} \simeq 1.25 \text{ TeV} \quad (N_C = 4, N_F = 8), \quad (103)$$

which is in accord with the above rough estimate $F_\phi \sim \sqrt{N_C N_F} m_F = \sqrt{4 \times 8} v_{\text{EW}}$. Amazingly, this value of F_ϕ turned out to be consistent with the current LHC Higgs data [22], as we shall discuss later.

In passing, the TD potential in $\mathcal{L}_{(2)\text{anomaly}}^S$ of Eq.(86) (with m_ϕ denoted as M_ϕ here) written in terms $\chi = e^{\phi/F_\phi}$ is rewritten in the TD field ϕ as [21]

$$V(\phi) = -\mathcal{L}_{(2)\text{anomaly}}^S = -\frac{M_\phi^2 F_\phi^2}{16} + \frac{1}{2} M_\phi^2 \phi^2 + \frac{4}{3} \frac{M_\phi^2}{F_\phi} \phi^3 + 2 \frac{M_\phi^2}{F_\phi^2} \phi^4 + \dots \quad (104)$$

It is remarkable to notice that in the anti-Veneziano limit the TD self couplings (trilinear and quartic couplings) are highly suppressed:

$$\frac{4}{3} \frac{M_\phi^2}{F_\phi} \sim \frac{1}{\sqrt{N_F N_C}}, \quad 2 \frac{M_\phi^2}{F_\phi^2} \sim \frac{1}{N_F N_C} \quad (105)$$

by $M_\phi/F_\phi \sim 1/\sqrt{N_F N_C}$ and $M_\phi \sim N_F^0 N_C^0$. It is also interesting to numerically compare the TD self couplings for the one-family model ($N_F = 8, N_C = 4$) having $v_{\text{EW}}/F_\phi \simeq 1/5$ with the self couplings of the SM Higgs with $m_h = M_\phi$, by making the ratios:

$$\begin{aligned} \left. \frac{g_{\phi^3}}{g_{h_{\text{SM}}^3}} \right|_{M_\phi=m_h} &= \left. \frac{\frac{4M_\phi^2}{3F_\phi}}{\frac{m_h^2}{2v_{\text{EW}}}} \right|_{M_\phi=m_h} \simeq \frac{8}{3} \left(\frac{v_{\text{EW}}}{F_\phi} \right) \simeq 0.5, \\ \left. \frac{g_{\phi^4}}{g_{h_{\text{SM}}^4}} \right|_{M_\phi=m_h} &= \left. \frac{\frac{2M_\phi^2}{F_\phi^2}}{\frac{m_h^2}{8v_{\text{EW}}^2}} \right|_{M_\phi=m_h} = 16 \left(\frac{v_{\text{EW}}}{F_\phi} \right)^2 \simeq 0.6. \end{aligned} \quad (106)$$

This shows that the *TD self couplings, although generated by the strongly coupled interactions, are even smaller than those of the SM Higgs*, a salient feature of the approximate scale symmetry in the anti-Veneziano limit. This is in sharp contrast to the widely-believed folklore, ‘‘Strong coupling solutions like Technicolor tend to lead to a strongly coupled Higgs’’ [26], as noted in the Introduction.

Finally, we should stress that the above estimated TD mass is stable against the feedback effects of the ETC (G_c term) through particularly the top quark loop, because of the large $F_\phi \simeq 5v_{\text{EW}}$. The loop corrections at the effective theory level including the SM sector and ETC effects were estimated to be [21]

$$\frac{\delta M_\phi^2|_{\phi^4}}{M_\phi^2} \simeq 24 \frac{m_F^2}{(4\pi F_\phi)^2} \simeq 6 \times 10^{-3}, \quad \frac{\delta M_\phi^2|_{\text{ETC/Yukawa}}}{M_\phi^2} \simeq -12(3 - \gamma_m)^2 \frac{m_F^2}{(4\pi F_\phi)^2} \frac{m_t^2}{M_\phi^2} \simeq -(3 - \gamma_m)^2 \frac{\delta M_\phi^2|_{\phi^4}}{M_\phi^2}, \quad (107)$$

which cancels each other as $\delta M_\phi^2/M_\phi^2 \approx 0$ for $\gamma_m = 2$ (see the comments of the last section), and are within 1% corrections to M_ϕ ($\delta M_\phi^2/M_\phi^2 \simeq -1.8 \times 10^{-2}$) even for $\gamma_m = 1$. Other loop effects are negligibly small.

BR[%]	gg	bb	WW	ZZ	$\tau\tau$	$\gamma\gamma$	$Z\gamma$	$\mu\mu$
$\Gamma_{\text{tot}} = 1.15 \text{ MeV}$	75.1	19.6	3.56	0.38	1.19	0.068	0.0048	0.0042

TABLE I: The TD branching ratios at 125 GeV in the one-family model with $N_C = 4$. The total width is also given.

V. THE LHC PHENOMENOLOGY OF TECHNIDILATON

Now that we have established ladder estimate of the mass and the decay constant of the TD, Eq.(102) and Eq.(103), we now discuss up-dating the previous analyses of the TD [22, 23] in view of the latest LHC data of the 125 GeV Higgs.

One can obtain the TD couplings to the SM gauge bosons and the SM fermions just by scaling from the SM Higgs as $v_{\text{EW}} \rightarrow F_\phi$ [21, 22]:

$$\begin{aligned} \frac{g_{\phi WW/ZZ}}{g_{\text{SM} WW/ZZ}} &= \frac{g_{\phi ff}}{g_{\text{SM} ff}} \quad (\text{for } f = t, b, \tau) \\ &= \frac{v_{\text{EW}}}{F_\phi} \left[\simeq \frac{1}{5} \ll 1 \quad (N_F = 8, N_C = 4) \right]. \end{aligned} \quad (108)$$

On the other hand, in the one-family model with $N_F = 8$ the couplings to digluon and diphoton include the colored/charged techni-fermion loop contributions along with a factor N_C [21, 22],

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\gamma\gamma, gg} &= \frac{\phi}{F_\phi} \left\{ \frac{\beta_F(g_s)}{2g_s} G_{\mu\nu}^2 + \frac{\beta_F(e)}{e} F_{\mu\nu}^2 \right\}, \\ \beta_F(g_s) &= \frac{g_s^3}{(4\pi)^2} \frac{4}{3} N_C, \quad \beta_F(e) = \frac{e^3}{(4\pi)^2} \frac{16}{9} N_C, \end{aligned} \quad (109)$$

where the beta functions have been evaluated at the one-loop level. Thus one finds the scaling from the SM Higgs [22] (Detailed formulae are given in the Appendix of Ref.[21]) ^{#19},

$$\begin{aligned} \frac{g_{\phi gg}}{g_{\text{SM} gg}} &\simeq \frac{v_{\text{EW}}}{F_\phi} \cdot (1 + 2N_C), \\ \frac{g_{\phi\gamma\gamma}}{g_{\text{SM}\gamma\gamma}} &\simeq \frac{v_{\text{EW}}}{F_\phi} \cdot \left(\frac{63 - 16}{47} - \frac{32}{47} N_C \right), \end{aligned} \quad (110)$$

where in estimating the SM contributions we have incorporated only the top (the terms of 1 and 16/47 for gg and $\gamma\gamma$ rates, respectively) and the W boson (the term of 63/47 for $\gamma\gamma$ rate) loop contributions. In Table I the branching fractions for relevant decay channels of the TD at 125 GeV are listed in the case of the one-family model with $N_C = 4$. Note that the total width $\Gamma_{\text{tot}} = 1.15 \text{ MeV}$ is smaller than the SM Higgs, which reflects the weaker couplings than the those of the latter, in contrast to the widely spread folklore mentioned in the Introduction.

Calculating the signal strengths for the LHC production categories (gluon gluon fusion (ggF), vector boson fusion (VBF), vector boson associate production (VH) and top associate production (ttH)),

$$\mu_{X_1 X_2}^i = \frac{\sigma_\phi^i \times \text{BR}(\phi \rightarrow X_1 X_2)}{\sigma_{\text{hSM}}^i \times \text{BR}(h \rightarrow X_1 X_2)}, \quad (111)$$

as a function of the overall coupling v_{EW}/F_ϕ for given the number of N_C , we may fit the $\mu_{X_1 X_2}^i$ to the latest data on the Higgs coupling measurements [80–91]. to determine the best-fit value of v_{EW}/F_ϕ . The result of the goodness of fit is shown in Table II, which updates the analysis in Ref. [22]. The Table II shows that the TD in the one-family model with $N_C = 4$ is favored by the current LHC Higgs data as much the same level as the SM Higgs. Remarkably, the best fit value $[v_{\text{EW}}/F_\phi]_{\text{best}} \simeq 0.2$, i.e. $F_\phi \simeq 5v_{\text{EW}}$ for $N_C = 4$ is in excellent agreement with the ladder estimate of the TD mass $\simeq 125 \text{ GeV}$ in Eq.(103)!

^{#19} One might think that the QCD interaction, which could be significant for the technidilaton, could spoil the walking picture based on the fixed point structure in the underlying one-family walking-technicolor dynamics. However, it is not the case since the QCD coupling is extremely small in magnitude for the energy region relevant to the walking technicolor dynamics, and the fixed point structure should be more sensitive to the higher-loops of the pure technicolor dynamics [76] rather than the QCD effects, if any.

N_C	$[v_{EW}/F_\phi]_{\text{best}}$	$\chi^2_{\text{min}}/\text{d.o.f.}$
3	0.27	25/17 \simeq 1.5
4	0.23	16/17 \simeq 0.92
5	0.17	32/17 \simeq 2.0
0 [SM Higgs]	1	8.0/18 \simeq 0.44

TABLE II: The best fit values of v_{EW}/F_ϕ for the one-family model with $N_C = 3, 4, 5$ displayed together with the minimum of the χ^2 (χ^2_{min}) normalized by the degree of freedom. Also has been shown in the last column the case of the SM Higgs corresponding to $N_C = 0$ and $v_{EW}/F_\phi = 1$.

In Table III we also make a list of the predicted signal strengths for each production category for the best fit value of $v_{EW}/F_\phi \simeq 0.23$ in the case with $N_C = 4$, along with the latest result reported from the ATLAS and CMS experiments [80–91]. Note the TD signal strengths in the dijet category (VBF), which involves the contamination by about 30% from the ggF + gluon jets, $gg \rightarrow \phi + gg$. The contribution from the ggF is highly enhanced compared to the SM Higgs case, due to the extra techni-quark loop contribution, which compensates the overall suppression by the direct VBF coupling $v_{EW}/F_\phi \simeq 0.2$ to lift the event rate up to be comparable to the SM Higgs case. (The detailed estimate of the ggF contamination is given in Appendix C.) Note also the suppression of the VH- $b\bar{b}$ -channel, which would be the characteristic signature of the TD to be distinguishable from the SM Higgs. More data from the upcoming LHC Run-II will draw a conclusive answer to whether or not the LHC Higgs is the SM Higgs, or the TD.

TD signal strengths ($v_{EW}/F_\phi = 0.23, N_C = 4$)	ATLAS	CMS
$\mu_{\gamma\gamma}^{\text{ggF}} \simeq 1.4$	1.32 ± 0.38	1.13 ± 0.35
$\mu_{ZZ}^{\text{ggF}} \simeq 1.0$	1.7 ± 0.5	0.83 ± 0.28
$\mu_{WW}^{\text{ggF}} \simeq 1.0$	0.98 ± 0.28	0.72 ± 0.37
$\mu_{\tau\tau}^{\text{ggF}} \simeq 1.0$	2.0 ± 1.4	1.1 ± 0.46
$\mu_{\gamma\gamma}^{\text{VBF}} \simeq 0.87$ (0.019)	0.8 ± 0.7	1.16 ± 0.59
$\mu_{ZZ}^{\text{VBF}} \simeq 0.61$ (0.014)	0.3 ± 1.3	1.45 ± 0.76
$\mu_{WW}^{\text{VBF}} \simeq 0.61$ (0.014)	1.28 ± 0.51	0.62 ± 0.53
$\mu_{\tau\tau}^{\text{VBF}} \simeq 0.61$ (0.014)	1.24 ± 0.57	0.94 ± 0.41
$\mu_{bb}^{\text{VH}} \simeq 0.014$	0.52 ± 0.40	1.0 ± 0.50

TABLE III: The predicted signal strengths of the TD with $v_{EW}/F_\phi = 0.23$ in the case of the one-family model with $N_C = 4$. The numbers in the parentheses correspond to the amount estimated without contamination from the ggF process. Also have been displayed the latest data on the Higgs coupling measurements reported from the ATLAS and CMS experiments [80–91].

The ATLAS and CMS have made a plot of the LHC Higgs couplings to the SM particles against the SM particle masses [92], shown that the LHC Higgs couplings to fermions have aligned very well with the SM Higgs boson properties. The plot has been made by assuming no contributions beyond the SM in loops, i.e., no contributions beyond SM to diphoton and digluon couplings. However, as explicitly seen from Eq.(110), the technidilaton couplings to diphoton and digluon significantly include the terms beyond the SM, technifermion contributions charged under the $U(1)_{\text{em}}$ or QCD color. In this respect, such a plot cannot be applied to the technidilaton. In fact, the successful consistency with the LHC Higgs coupling measurement, as shown in Table III, is due to those beyond SM contributions, which especially enhance the ggF production cross section, balanced by the overall suppression due to the coupling F_ϕ larger than v_{EW} by a factor of 5.

VI. BEYOND TECHNIDILATON: OTHER TECHNIHADRONs?

As we discussed in subsection II E (see discussions below Eq.(65)), other techni-hadron (techni- ρ , techni- a_1 , technibaryon, etc.) masses also have masses of order, $M_A = \mathcal{O}(C_A(r)m_F) = C_A(r) \cdot \Lambda_{\text{TC}} \cdot f\left(\frac{\alpha}{\alpha_{\text{cr}}}\right) (\ll \Lambda_{\text{TC}})$, with the universal scaling of Miransky-BKT type, $f\left(\frac{\alpha}{\alpha_{\text{cr}}}\right) \sim \tilde{f}(r)$, up to the non-universal coefficient $C_A(r)$ depending on the each techni-hadron A , with possible dependence on $r = N_F/N_C$ in the anti-Veneziano limit. This is in sharp contrast to the ordinary QCD where $F_\pi = \mathcal{O}(m_F) = \mathcal{O}(\Lambda_{\text{QCD}})$. The TD as a pseudo NG boson has the mass solely due to the

explicit breaking of the scale symmetry via the PCDC just in the same way as the pion does. As mentioned above, the TD mass $M_\phi = \mathcal{O}(m_F/2)$ is independent of N_C, N_F as the PCDC relation dictates.

In contrast, all the non-NG boson techni-hadrons have *no constraints from the PCDC as the explicit breaking of the scale symmetry* but do have *constraints from the SSB of the scale symmetry*, so that they should have masses on the scale of the SSB of the scale symmetry, characterized by $F_\phi \sim \sqrt{N_F N_C} m_F$ which is much larger than $2m_F$ of the naive nonrelativistic quark model picture, particularly in the anti-Veneziano limit of the walking case, $N_C \rightarrow \infty$ with $\lambda \equiv N_C \alpha = \text{constant}$ ($\alpha > \alpha_{\text{cr}}$), and with $r \equiv N_F/N_C = \text{constant}$ ($\gg 1$). We naturally expect that their masses are generally of order of $\mathcal{O}(\text{TeV's})$:

$$M_A = \mathcal{O}(C_A(r) m_F) = \mathcal{O}(\text{TeV's}) \gg 2m_F \gg M_\phi \quad (112)$$

with $C_A(r) \gg 1$ for $r \rightarrow r_{\text{cr}}$. This is consistent with the straightforward computation of large N_F QCD based on the ladder BS equation combined with the ladder SD equation, $M_\rho \simeq 4m_F \simeq 12F_\pi$ (for $N_C = 3$) [28, 29] (This corresponds to $\simeq 6v_{\text{EW}}$ in the one-family model with $N_F = 8, N_C = 4$), which is somewhat larger than the QCD case $M_\rho \sim 8F_\pi$. Being highly strong-coupling relativistic result, it is contrasted to the naive weak-coupling non-relativistic quark-model view $M_A \sim 2m_F$. This is also compared with the N_C counting of the bound state masses $\mathcal{O}(\Lambda_{\text{QCD}})$ in the ordinary QCD, where the gluon loop is dominant, while *the fermion loop dominates in the anti-Veneziano limit in the walking theory*. Also the holographic calculations tend to give $M_A \gg M_\phi$, and so do the recent lattice calculations [38, 93].

As usual, the IR conformal physics of the WTC should be described by the low-lying composite fields as effective fields, in a way to realize all the symmetry structure of the underlying theory. Such an effective theory of WTC having higher resonances together with the 125 GeV TD is already constructed as a straightforward extension of sChPT [21, 27], i.e., the scale-invariant version [30] of the Hidden Local Symmetry (HLS) model [55, 56], (the “sHLS model”), where the technirho mass terms have the scale-invariance non-linearly realized by the TD field $\chi = e^{\phi/F_\phi}$, with the SSB of the scale invariance characterized by the scale of F_ϕ , while the Higgs (TD) mass term in the TD potential, on the order of $m_F (\ll F_\phi)$, is the only source of the explicit breaking of the scale symmetry related (via PCDC) to the nonperturbative trace anomaly of the underlying theory.

Interesting candidate for such techni-hadrons may account for the diboson excesses recently observed at LHC at 2 TeV [57, 58], which can be identified with the walking technirho [59]. A smoking gun of the walking technirho is the absence of the decay to the 125 GeV Higgs (TD), which is forbidden by the scale symmetry explicitly broken only by the Higgs (TD) mass term (corresponding to the nonperturbative trace anomaly in the underlying WTC) [60]. Actually, the salient feature of the scale symmetry of the generic effective theory not just the sHLS model, containing the SM gauge bosons and the Higgs plus new vector bosons (any other massive particles as well), is the absence of the decay of the new vector bosons such as the technirho (and also other higher resonances) into the 125 GeV Higgs plus the SM gauge bosons [60]. If such a decay of new particles is not found at LHC Run II, then the 125 GeV Higgs is nothing but the dilaton (TD in the case of the WTC) responsible for the *nonlinearly realized* scale symmetry, i.e., the SSB of the scale symmetry, no matter what underlying theory may be beyond the SM. This should be tested in the future LHC experiments.

VII. SUMMARY AND DISCUSSIONS

In conclusion we have shown that the technidilaton in the walking technicolor, typically realized in the one-family model ($N_F = 8, N_C = 4$), is a naturally light composite Higgs to be identified with the 125 GeV Higgs, and is a weakly coupled composite state out of the strongly coupled conformal gauge dynamics, with its each coupling being even weaker than the SM Higgs.

In this paper, the walking technicolor with $\gamma_m = 1$, originally based on the ladder SD equation, is reformulated in terms of the Caswell-Banks-Zaks infrared fixed point α_* of the $SU(N_C)$ gauge theory for N_F massless flavors, with the intrinsic scale Λ_{TC} , in the anti-Veneziano limit Eq.(1):

$$N_C \rightarrow \infty, \lambda \equiv N_C \alpha = \text{fixed}, \text{ with } r \equiv N_F/N_C = \text{fixed} \gg 1, \quad (113)$$

where the input coupling in the SD kernel is given by the constant coupling Eq.(21) and Fig. 2:

$$\alpha(x) = \alpha_* \theta(\Lambda_{\text{TC}}^2 - x), \quad (x = -p^2 > 0). \quad (114)$$

We have shown in the anti-Veneziano limit that the SSB of the chiral (electroweak) and scale symmetries takes place due to the technifermion condensate in Eq.(55), $\langle \bar{F}F \rangle_R \simeq -\frac{N_C}{\pi^2} m_F^3$, at strong coupling, $\alpha = \alpha_* > \alpha_{\text{cr}} (r < r_{\text{cr}})$, in

such a way that it is essentially a continuous phase transition at criticality $r = r_{\text{cr}}$ as the conformal phase transition characterized by the Miransky-BKT scaling of the essential singularity, Eq.(44):

$$m_F \sim \Lambda_{\text{TC}} \cdot \exp \left(-\frac{\pi}{\sqrt{\frac{\alpha}{\alpha_{\text{cr}}} - 1}} \right) \ll \Lambda_{\text{TC}} \quad \left(0 < \frac{\alpha}{\alpha_{\text{cr}}} - 1 = \frac{\alpha_*}{\alpha_{\text{cr}}} - 1 \sim (r_{\text{cr}} - r) \ll 1 \right). \quad (115)$$

Here the CBZ IR fixed point (viewed from $\mu > \Lambda_{\text{TC}}$) is now regarded as the UV fixed point (viewed from $\mu < \Lambda_{\text{TC}}$). The corresponding nonperturbative beta function has a multiple zero with the zero curvature at $\alpha(\mu) = \alpha_* = \alpha_{\text{cr}}$ as in Eq. (45), where the coupling turns over to the weak coupling region $\alpha(\mu) < \alpha_* = \alpha_{\text{cr}}$. See Fig.3.

Accordingly, while there are no bound states in the conformal window $\alpha < \alpha_{\text{cr}}$ (unparticle phase), bound states exist only in the SSB phase, all of order m_F up to the factors depending on $r = N_F/N_C$ in the anti-Veneziano limit.

First, the pseudo NG boson masses come only from explicit breakings of the internal symmetry, the chiral $SU(8)_L \times SU(8)_R$ in the car of one-family model, through the Dashen formula. The technipions (uneaten pseudo NG bosons of the chiral symmetry) pick up the explicit breaking of $SU(8)_L \times SU(8)_R$ by the SM gauge interactions and ETC gauge interactions,

$$M_\pi^2 = \langle \pi | \mathcal{H}_{\text{breaking}} | \pi \rangle = \frac{1}{F_\pi^2} \langle 0 | \delta_\pi \delta_\pi \mathcal{H}_{\text{breaking}} | 0 \rangle, \quad \mathcal{H}_{\text{breaking}} = g_{(SM/ETC)}^2 \int d^4x D_{\mu\nu}^{(SM/ETC)}(x) T \left(J_{(SM/ETC)}^\mu(x) J^\nu(0)_{(SM/ETC)} \right). \quad (116)$$

with enhancement due to the large anomalous dimension, where $\delta_\pi O \equiv [iQ_{5\pi}, O]$, with the broken generator charge $Q_{5\pi}$ corresponding to the π , and $D_{\mu\nu}^{(SM/ETC)}$ is the SM/ETC gauge boson propagator coupled to the source current $J_{(SM/ETC)}^\mu$. They all have mass of $\gtrsim \mathcal{O}(m_F)$, see Eq.(60).

Similarly, the technidilaton, the pseudo NG boson of the scale symmetry, acquires the mass from the explicit breaking of the scale symmetry, m_F , since the SSB of the scale symmetry due to the mass generation of m_F also breaks the scale symmetry explicitly. The mass is also evaluated through the Dashen formula for the nonperturbative trace anomaly, Eq.(77), this time the PCDC relation, Eq.(88):

$$M_\phi^2 = \langle \phi | \mathcal{H}_{\text{breaking}} | \phi \rangle = \langle \phi | \frac{1}{4} \theta_\mu^\mu | \phi \rangle = \frac{1}{4F_\phi^2} \langle 0 | \delta_D \delta_D \theta_\mu^\mu | 0 \rangle \\ = \frac{1}{F_\phi^2} \cdot \langle 0 | 4 \theta_\mu^\mu | 0 \rangle \sim \frac{1}{N_F N_C m_F^2} \cdot \left(N_F N_C \frac{16\xi^2}{\pi^4} m_F^4 \right) \simeq \left(\frac{m_F}{2} \right)^2 \ll F_\phi^2, \quad (117)$$

where $\delta_D \theta_\mu^\mu = d_\theta \theta_\mu^\mu$ ($d_\theta = 4$) is the dilatation transformation. Note that the technidilaton decay constant F_ϕ is the order parameter of the SSB of the scale symmetry, $F_\phi^2 \sim N_F N_C m_F^2$ by definition, while the *explicit breaking scale* is m_F which is much smaller than the SSB scale F_ϕ of the scale symmetry in the anti-Veneziano limit (see text).

We have particularly seen that the nonperturbative trace anomaly is RG-invariant:

$$\langle \theta_\mu^\mu \rangle = \frac{\beta^{(NP)}(\alpha(\mu))}{4\alpha(\mu)} \langle G_{\mu\nu}^2 \rangle_{(\mu)}^{\text{NP}} = \mu - \text{independent}, \quad (118)$$

in a way that the techni-gluon condensate is enhanced by the anomalous dimension as in Eq.(73), which is precisely compensated as the vanishing beta function Eq.(74), finally to arrive at the RG-invariant finite result as in Eq.(75). Thus the small beta function near the criticality is only operative for the large hierarchy $m_F \ll \Lambda_{\text{TC}}$, while further hierarchy $M_\phi \ll m_F \ll F_\phi$ is due to the anti-Veneziano limit ^{#20}:

$$\frac{M_\phi}{F_\phi} \sim \frac{m_F}{F_\phi} \sim \frac{1}{\sqrt{N_F N_C}} \rightarrow 0. \quad (119)$$

It is a salient feature of the anti-Veneziano limit that the technidilaton has a limit of vanishing mass in units of F_ϕ , Eq.(98), though not the exact massless point at the criticality of the conformal phase transition point where no bound states exist for the exactly zero explicit breaking $m_F \equiv 0$, i.e., no nonperturbative trace anomaly (see text).

^{#20} Note that other explicit breakings, quark/lepton mass $m_{q/l}$ (Eq.(51)), technipion mass M_π (Eq.(60)), also scale like $m_{q/l}/F_\phi$, $M_\pi/F_\phi \sim 1/\sqrt{N_F N_C} \rightarrow 0$ in the anti-Veneziano limit. They have an exactly massless point (switching off the ETC gauge interaction), in contrast to the technidilaton, though.

This is somewhat similar to the η' meson in ordinary QCD, which is regarded as the pseudo NG boson having mass from the $U(1)_A$ anomaly, $M_{\eta'}/\Lambda_{\text{QCD}} \sim \frac{N_F}{N_C} \ll 1$ and $M_{\eta'}/F_\pi \sim \frac{\sqrt{N_F}}{N_C} \rightarrow 0$ in the original Veneziano limit ($r = N_F/N_C \ll 1$ instead of the anti-Veneziano limit $r \gg 1$). Fate of the η' in the anti-Veneziano limit was discussed in the text, see Eq.(101). The exact massless point is also absent for η' , since the anomaly cannot be identically zero in the quantum theory. Also note that the realistic value of the η' meson is far from light in the real-life QCD, which corresponds to the technipions in our case.

For the phenomenological issue of the technidilaton to be identified with the 125 GeV Higgs, we first noticed that the Pagels-Stokar formula for the weak scale v_{EW} in Eq.(96) implies $(246 \text{ GeV})^2 = v_{\text{EW}}^2 = N_F N_C \frac{\xi^2}{4\pi^2} m_F^2$. Then we have a conceptual feature of the technidilaton in the anti-Veneziano limit:

$$\frac{M_\phi}{v_{\text{EW}}} \sim \frac{M_\phi}{F_\phi} \sim \frac{1}{\sqrt{N_F N_C}} \rightarrow 0. \quad (120)$$

More quantitatively, we showed the ladder estimate of the PCDC relation together with the Pagels-Stokar formula leads to Eq.(102):

$$M_\phi^2 \simeq \left(\frac{v_{\text{EW}}}{2}\right)^2 \cdot \left(\frac{5 v_{\text{EW}}}{F_\phi}\right)^2 \cdot \left[\frac{8}{N_F} \frac{4}{N_C}\right]. \quad (121)$$

Hence in the particular case, the one-family model with $N_F = 8, N_C = 4$, we have $m_F \simeq v_{\text{EW}}$ and in fact realize the reality of 125 GeV Higgs as in Eq.(103):

$$M_\phi \simeq \frac{v_{\text{EW}}}{2}, \quad F_\phi \simeq 5 v_{\text{EW}} \quad (N_F = 8, N_C = 4). \quad (122)$$

The result yields in fact a best fit to the current LHC data for the 125 GeV Higgs as was explained in details in Section V. See Table II and III. The couplings of the technidilaton to the SM particles are small by a factor of $\frac{v_{\text{EW}}}{F_\phi} \simeq \frac{1}{5}$, which is compensated by the enhancement by the extra contributions from the charged/colored technifermions in the one-family model, see Eqs.(108) – (110). Then the net results happened to be similar to that of the SM Higgs within the errors of the current data at LHC.

As to the non-NG boson technihadons, $\{A\}$, they all have the mass M_A characterized by the coefficient $C_A(r)$ depending on the ratio $r = N_F/N_C$ in the anti-Veneziano limit:

$$M_A \gtrsim \mathcal{O}(C_A(r) m_F) \gg m_F \gtrsim M_\phi, \quad (123)$$

which takes the form of the universal scaling of essential singularity: $M_A \sim C_A(r) \Lambda_{\text{TC}} \cdot e^{-\pi/\sqrt{\alpha/\alpha_{\text{cr}}-1}} \rightarrow 0$ ($\alpha/\alpha_{\text{cr}} \rightarrow 1$, or $r \rightarrow r_{\text{cr}}$). In the anti-Veneziano limit at $\alpha \rightarrow \alpha_{\text{cr}}$ we have

$$\frac{M_\phi}{M_A} \sim \frac{1}{C_A(r)} \ll 1, \quad \frac{M_A}{M_B} \rightarrow \frac{C_A(r)}{C_B(r)} \neq 0, \infty. \quad (124)$$

Interesting candidate for such techni-hadrons may explain the diboson excesses recently observed at LHC at 2 TeV [57, 58], which can be identified with the walking technirho with $M_\rho \simeq 2 \text{ TeV}$ [59]. The excesses suggest a characteristically small width $\Gamma_{\text{total}} < 100 \text{ GeV}$ [57], which can be naturally realized in the anti-Veneziano limit:

$$\frac{\Gamma_{\text{total}}}{M_\rho} \simeq \frac{\Gamma(\rho \rightarrow WW/WZ)}{M_\rho} \simeq \frac{1}{48\pi} \frac{g_{\rho\pi\pi}^2}{N_D} \sim \frac{1}{N_F N_C} \rightarrow 0, \quad (125)$$

where $N_D = N_F/2$ is the number of the weak-doublets. In fact our one-family model $N_F = 8, N_C = 4$ can reproduce the features of the excesses very well [59]. The fact that $\Gamma_{\text{total}} \simeq \Gamma(\rho \rightarrow WW/WZ)$ is related to a smoking gun of the walking technirho, namely the absence of the decay to the 125 GeV Higgs (TD), which is forbidden by the scale symmetry explicitly broken only by the Higgs (TD) mass term (corresponding to the nonperturbative trace anomaly in the underlying WTC) [60]. Actually, it was shown [60] that the salient feature of the scale symmetry of the generic effective theory not just the sHLS model, containing the SM gauge bosons and the Higgs plus new vector bosons (any other massive particles as well), is the absence of the decay of the new vector bosons into the 125 GeV Higgs plus the SM gauge bosons, invalidating the so-called ‘‘Equivalence Theorem’’. It was further shown that if such a decay of new particles is not found at LHC Run II, then the 125 GeV Higgs is nothing but the dilaton (technidilaton in the case of the WTC) responsible for the *nonlinearly realized* scale symmetry, i.e., the *SSB* of the scale symmetry, no matter whatever underlying theory may be beyond the SM. This should definitely be tested in the future LHC experiments. We will see.

Several comments are in order:

1. The S parameter: The ladder BS calculation of the S parameter from the techni-sector alone was done near the criticality [70, 94], suggesting a sizable reduction, up till the 40% reduction per one weak doublet compared with the QCD. Including a (weak) ETC effects among technifermions (G_b terms in Eq.(50)) for $\alpha/\alpha_{\text{cr}} > 1$ in the ladder BS calculation further reduces it to $\frac{S}{N_F N_C} \simeq 0.03$, which would imply $S \simeq 1$ for the one-family model ($N_F = 8$, $N_C = 4$). This is still in conflict with the bound from electroweak precision experiments, $S < 0.1$. The S parameter from the TC sector, however, is not necessarily in conflict with the experimental value of the S from the electroweak precision measurements, since the contributions from the TC sector can easily be cancelled by the strong mixing with the SM fermion contribution through the ETC interactions [41] (G_c terms in Eq.(50)), as in the fermion delocalization of the Higgsless model [95]. the inclusion of the strong ETC effects plus the induced four-fermion effects of WTC, and/or the strong ETC mixing effects between the technicolor and SM sectors (G_c terms in Eq.(50)). This should be studied explicitly in the ladder BS equation. Moreover, even within the TC sector alone, there exists a way to resolve this problem as demonstrated in the holographic model [23, 31], where we can reduce S by tuning the holographic parameter of strength of the techni-gluon condensate G through the z_m (position of the infrared brane) in a way consistent with the TD mass of 125 GeV and all the current LHC data for the 125 GeV Higgs [23]. This approach can be constrained from the technipion mass bound from experiments [32]. Such a large gluonic effects cannot be incorporated into the ladder calculations in principle, and should be checked by the lattice calculations. The more straightforward calculations on the lattice are highly desired anyway.

2. The top quark mass: The top mass is too small for the anomalous dimension $\gamma_m = 1$. There are possible resolutions: First, the ETC breaking takes place in a step-wise, with the smallest scale for the third family $\Lambda_3 \ll \Lambda_2 \ll \Lambda_1$, and Λ_3 is less constrained by the FCNC limit than $\Lambda_{2,1} (> 10^3 \text{ TeV})$, as commonly used in the ETC model building [17, 96]. Another way out [9, 73] is the even larger anomalous dimension $1 < \gamma_m = 1 + \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}} < 2$ [9] in the ladder calculation of the gauged NJL model, Eq.(62), due to the strong ETC coupling (G_b terms in Eq.(50)). Note that the G_a term for the third family in Eq.(50) is much stronger than those for the first and second families, and comparable to G_b, G_c terms at the scale Λ_3 . The strong G_a term for the top quark triggers the top quark condensate [3] as well as the technifermion condensate, with $\gamma_m \simeq 2$, but would lead to a different picture than the “top-colored assisted TC” [97], which had a serious problem of the light top pion not absorbed into W/Z [98], since the W/Z mass is already generated by the TC condensate. In the case at hand, along the critical line of the system together with almost comparable G_b, G_c terms, only a single combination of the top and technifermions may condense, so that no extra NG bosons would be formed.

3. Straightforward ladder BS calculations: The walking techni-hadrons spectra by the BS and SD equations were calculated for scalar, vector, axialvector mesons: M_S, M_ρ, M_{a_1} , together with the decay constants G_S, F_ρ, F_{a_1} [28]. The result shows $M_S/f_\pi \sim 4$, $M_\rho/F_\pi \simeq M_{a_1}/F_\pi \simeq 12$, which is compared with the real-life QCD, $m_{f_0(1370)}/f_\pi \simeq 15$ ($m_{f_0(500)}/f_\pi \simeq 5$: may not be $\bar{q}q$ bound state), $m_\rho/f_\pi \simeq 8$, $m_{a_1}/f_\pi \simeq 13$. The near degeneracy $M_\rho \simeq M_{a_1}$ is also consistent with the lattice results for $N_F = 8$, $N_C = 3$ [38, 93]. Since the calculation does not distinguish between the flavor singlet and nonsinglet states, the scalar state S does not corresponds to the technidilaton as a flavor singlet $\bar{F}F$ bound state. Nevertheless, it would be suggestive that the scalar state S has the mass much smaller than in the QCD. It is well known [99] that the singlet-nonsinglet splitting can be done by introducing the Kobayashi-Maskawa-’t Hooft determinant [100] arising from the instanton in such a way as to push the flavor-singlet scalar down and nonsinglet up. It would be interesting to see the same thing near criticality of the walking theories in the ladder BS equation. Another interesting feature of the result of [28] is that $F_\rho/F_\pi \simeq 2$ compared with the QCD value $F_\rho/f_\pi \simeq \sqrt{2}$, which could be relevant to the 2 TeV diboson excesses at the LHC [59].

4. One-family model on the lattice: We have shown that the ladder results for the one-family model with $N_F = 8$, $N_C = 4$ give the technidilaton as the 125 GeV Higgs the best fit to the current LHC data. The holographic estimate also yields a similar result as far as the realistic point is concerned [23]. It was further shown that a natural setting of the ETC model building prefers $N_C = 4$. Although many lattice results indicate the walking behavior with $\gamma_m \simeq 1$ [37–39] and a light flavor-singlet as a candidate for the technidilaton [33] in the $N_F = 8$, $N_C = 4$ theory, so far no lattice studies for $N_F = 8$, $N_C = 4$ have been done. Lattice results for $N_F = 8$, $N_C = 4$ are highly desired.

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Appendix A: Ladder estimate of the chiral condensate, anomalous dimension, and OPE

The bare chiral condensate can be directly estimated from the SSB solution Eq.(42) at $x = \Lambda^2$ with the UV boundary condition Eq.(43), $\tilde{\omega} \ln(16\Lambda^2/m_F^2) \simeq \pi$, for $m_0 = 0$. It follows:

$$\Lambda^2 \Sigma(\Lambda) = -\Lambda^4 \Sigma'(\Lambda) \simeq \frac{2\xi}{\pi \tilde{\omega}} \Lambda m_F^2 \sin(\tilde{\omega} \ln(16\Lambda^2/m_F^2) - 2\tilde{\omega}) \simeq \frac{4\xi}{\pi} \Lambda m_F^2, \quad (\text{A1})$$

which is also consistent with Eq.(47). Hence the chiral condensate can be evaluated through the formula Eq.(33) with $m_0 = 0$ (or Eq.(34)): [69]

$$\langle \bar{F}F \rangle_0 = -\frac{N_C}{\pi^2} \frac{\alpha_{\text{cr}}}{\alpha(\Lambda^2)} \Lambda^2 \Sigma(\Lambda) \simeq -\frac{4\xi N_C}{\pi^3} m_F^2 \Lambda, \quad (\text{A2})$$

without logarithmic factor, which in fact implies $Z_m^{-1} \propto \frac{\Lambda}{m_F}$, and hence

$$\gamma_m = \Lambda \frac{\partial}{\partial \Lambda} \ln Z_m^{-1} = 1 \quad (\text{A3})$$

consistently with Eq.(49) obtained in comparison with the OPE.

In the case of $m_0 \neq 0$, the result $Z_m \sim \frac{m_F}{\Lambda}$ was also noted in the SSB phase [24] before the advent of WTC, through the UV boundary condition. More specifically, using the expression for $m_0 \neq 0$ in Eq.(43) with $\Sigma(x = m_P^2) = m_P \simeq m_F + m_R$ for $m_R \ll m_F$, we have [69]

$$m_0 \equiv \varphi(m_P^2) = (x\Sigma(x))'|_{x=\Lambda^2} = \frac{\xi}{\pi \tilde{\omega}} \frac{m_P^2}{\Lambda} \sin\left(\tilde{\omega} \ln \frac{16\Lambda^2}{m_P^2}\right) \simeq \varphi(m_F^2) + m_R \frac{\partial \varphi(m_P)}{\partial m_P} \Big|_{m_P=m_F} \quad (\text{A4})$$

$$\simeq m_R \left(\frac{2\xi}{\pi} \frac{m_F}{\Lambda} \right), \quad (\text{A5})$$

$$Z_m \simeq \frac{2\xi}{\pi} \frac{m_F}{\Lambda}, \quad (\text{A6})$$

which agrees with the result obtained directly performing the integral Eq.(31) [69]. This again yields the same result as Eq.(A3), $\gamma_m = 1$. From Eq.(A6) we have the renormalized condensate at $\mu = m_F$:

$$\langle \bar{F}F \rangle_R = \langle \bar{F}F \rangle_0 Z_m \simeq -\frac{8\xi^2 N_C}{\pi^4} m_F^3, \quad (\text{A7})$$

so that the multiplicative renormalization follows, i.e., $m_0 \langle \bar{F}F \rangle_0 = m_R \langle \bar{F}F \rangle_R$.

This large anomalous dimension provides an enhancement of the quark/lepton mass through the ETC having the scale $\Lambda = \Lambda_{\text{ETC}} (\sim \Lambda_{\text{TC}})$:

$$m_{q/l} \sim -c_i \frac{\langle \bar{F}F \rangle_0}{N_C \Lambda_{\text{ETC}}^2} \simeq \left(c_i \frac{4\xi}{\pi^3} \frac{m_F^2}{\Lambda_{\text{ETC}}} \right) = \left[\frac{c_i}{N_F} \left(\frac{4}{\xi \pi} \frac{4v_{\text{EW}}}{N_C \Lambda_{\text{ETC}}} \right) \right] v_{\text{EW}} \equiv y^{\text{eff}} \cdot v_{\text{EW}}, \quad (\text{A8})$$

$$y^{\text{eff}} \sim \frac{c_i}{N_F} \left(\frac{4}{\xi \pi} \frac{4v_{\text{EW}}}{N_C \Lambda_{\text{ETC}}} \right), \quad (\text{A9})$$

where c_i is a model-dependent numerical constant of $\mathcal{O}(1)$ and the Pagels-Stokar formula $m_F^2 \simeq v_{\text{EW}}^2 \frac{4\pi^2}{N_F N_C}$ in Eq.(B4) was used. This is roughly 0.1 GeV for the typical quark/lepton mass (except for the top quark) with the effective Yukawa coupling $y^{\text{eff}} \lesssim 10^{-3}$, in accord with the FCNC constraints $\Lambda_{\text{ETC}} \gtrsim 10^3 \text{ TeV} \simeq 4v_{\text{EW}}$. Then the WTC with this ladder solution provides a concrete dynamics for a solution of the FCNC problem by the large anomalous dimension, the solution [13] based on a pure assumption of existence of a theory having UV fixed point (without concrete dynamics nor a concrete value of the anomalous dimension).

The asymptotic form of $\Sigma(x)$ ($m_0 \neq 0$) takes the same form as Eq.(42), with replacement of m_F to m_P .

$$\Sigma(x) \sim \frac{2\xi}{\pi} \frac{1}{\tilde{\omega}} \frac{m_P^2}{\sqrt{x}} \sin\left(\tilde{\omega} \ln\left(\frac{16x}{m_P^2}\right) - 2\tilde{\omega}\right) \simeq \frac{2\xi}{\pi} \frac{1}{\tilde{\omega}} \frac{m_P^2}{\sqrt{x}} \sin\left(\tilde{\omega} \ln\left(\frac{16x}{m_F^2}\right) - 2\tilde{\omega} \ln\left(1 - \frac{m_R}{m_F}\right) - 2\tilde{\omega}\right) \quad (\text{A10})$$

$$\simeq \frac{4\xi}{\pi} \frac{(m_F + m_R)^2}{\sqrt{x}} \left(1 - \frac{m_R}{m_F}\right) \simeq \frac{4\xi}{\pi} \left[\frac{m_F m_R}{\sqrt{x}} + \frac{m_F^2}{\sqrt{x}} \right] \quad (\text{A11})$$

This is perfectly consistent with the OPE with $\gamma_m = 1$:

$$\Sigma(x) \sim \frac{4\xi}{\pi} m_R \left(\frac{x}{m_F^2} \right)^{-\gamma_m/2} - \frac{\pi^3}{2\xi N_C} \frac{\langle \bar{F}F \rangle_R}{x} \left(\frac{x}{m_F^2} \right)^{\gamma_m/2} \sim \frac{4\xi}{\pi} \left[\frac{m_R m_F}{\sqrt{x}} + \frac{m_F^2}{\sqrt{x}} \right]. \quad (\text{A12})$$

Such a large anomalous dimension is due to the UV fixed point at α_{cr} whose effective coupling $C_2 \alpha_{\text{cr}} = \pi/3$ remains strong all the way up to the scale Λ_{TC} .

Appendix B: Pagels-Stokar Formula

The Pagels-Stokar formula [101] is given as

$$\frac{4\pi^2}{N_C} \frac{F_\pi^2}{m_F^2} = \int_0^{\Lambda^2 \rightarrow \infty} dx x \frac{\Sigma(x)^2 - \frac{x}{4} \frac{d(\Sigma(x)^2)}{dx}}{(x + \Sigma(x)^2)^2}. \quad (\text{B1})$$

The integral is dominated by the infrared region and converges quickly for the ladder SSB solution with $\gamma_m = 1$, $\Sigma(x) \sim m_F^2/\sqrt{x}$, and hence is insensitive to the value of Λ^2 as far as it is very large, say $\Lambda^2/m_F^2 > 10^6$. Rather, the integral depends on the precise form in the infrared region $x < \Lambda^2$. Here we take the mass function of the ladder SD solution, Eq.(42):

$$\Sigma(x) \simeq \xi \frac{m_F^2}{\sqrt{x}} \sqrt{\frac{\text{cth} \pi \tilde{\omega}}{\pi \tilde{\omega} (\tilde{\omega}^2 + 1/4)}} \sin \left(\tilde{\omega} \ln \left(\frac{16x}{m_F^2} \right) - 2\tilde{\omega} \right). \quad (\text{B2})$$

Then we get

$$\frac{4\pi^2}{N_C} \frac{F_\pi^2}{m_F^2} \simeq 2.00 \xi^2. \quad (\text{B3})$$

From this we get

$$v_{\text{EW}}^2 = N_D F_\pi^2 = \frac{N_F}{2} \frac{\xi^2}{2\pi^2} N_C m_F^2 \simeq \frac{\xi^2}{4\pi^2} N_F N_C m_F^2. \quad (\text{B4})$$

The result is compared with that obtained by using a naive mass function, $\Sigma(x) = m_F^2 x^{-1/2}$ for $x > m_F^2$ and $\Sigma(x) = m_F$ for $x < m_F^2$: $\frac{4\pi^2}{N_C} \frac{F_\pi^2}{m^2} \simeq 1.00$.

Appendix C: Estimate of ggF contamination for technidilaton production with forward dijet at LHC

The $h + 2j$ production at the LHC arises dominantly from two processes, i.e., VBF and ggF:

$$\sigma(h + 2j) = \sigma_{\text{VBF}}(h + qq) + \sigma_{\text{ggF}}(h + gg). \quad (\text{C1})$$

In Ref. [102] the $h + gg$ cross section has been estimated, at $\sqrt{s} = 14$ TeV with a kinematical cut set, as a function of the Higgs mass m_h . At $m_h = 125$ GeV it reads

$$\sigma_{\text{ggF}}^{\text{cut}}(h + gg) \simeq 10 \text{ pb}, \quad (\text{C2})$$

which is about 70% - 80% amount of the full phase space due to the kinematical cut. Taking into account the phase space cut one can evaluate the full cross section as

$$\begin{aligned} \sigma_{\text{ggF}}^{\text{full}}(h + gg) &\sim \left(\frac{10}{8} - \frac{10}{7} \right) \times \sigma_{\text{ggF}}^{\text{cut}}(h + gg) \\ &\sim 13 - 14 \text{ pb}. \end{aligned} \quad (\text{C3})$$

On the other hand, the $h + 0j$ ggF production cross section at $\sqrt{s} = 14$ TeV can be read off from Ref. [103] as

$$\sigma_{\text{ggF}}^{\text{full}}(h + 0j) \simeq 49 \text{ pb}, \quad (\text{C4})$$

which corresponds to the number obtained by integrating the full phase space. Eqs.(C3) and (C4) allow us to numerically write the ratio

$$r_{\text{ggF}+jj} \equiv \frac{\sigma_{\text{ggF}}^{\text{full}}(h+gg)}{\sigma_{\text{ggF}}^{\text{full}}(h+0j)} \simeq 0.3. \quad (\text{C5})$$

Note that the dependence of the gluon-gluon-Higgs coupling is canceled in this ratio, so the value of the ratio can be applied to any Higgs candidate including the technidilaton.

We shall assume that the 8 TeV cross sections are also applicable to Eq.(C5). Then the signal strength of the technidilaton decaying to $X_1 X_2$ through the dijet production channel can be evaluated as

$$\begin{aligned} \mu_{2j}^{X_1 X_2} &= \frac{\sigma(\phi+2j) \times \text{BR}(\phi \rightarrow X_1 X_2)}{\sigma(h+2j) \times \text{BR}(h \rightarrow X_1 X_2)} \\ &\sim \frac{[\sigma_{\text{VBF}}(\phi+qq) + r_{\text{ggF}+jj} \times \sigma_{\text{ggF}}(\phi+0j)] \times \text{BR}(\phi \rightarrow X_1 X_2)}{[\sigma_{\text{VBF}}(h+qq) + r_{\text{ggF}+jj} \times \sigma_{\text{ggF}}(h+0j)] \times \text{BR}(h \rightarrow X_1 X_2)} \\ &= R_{\text{VBF}} \cdot \frac{1 + r_{\text{contam}}(\phi)}{1 + r_{\text{contam}}(h)} \cdot r_{\text{BR}}^{X_1 X_2}, \end{aligned} \quad (\text{C6})$$

where

$$\begin{aligned} R_{\text{VBF}} &= \frac{\sigma_{\text{VBF}}(\phi+qq)}{\sigma_{\text{VBF}}(h+qq)} = \left(\frac{v_{\text{EW}}}{F_\phi} \right)^2, \\ r_{\text{contam}}(\phi/h) &= \frac{r_{\text{ggF}+jj} \times \sigma_{\text{ggF}}(\phi/h+0j)}{\sigma_{\text{VBF}}(\phi/h+qq)}, \\ r_{\text{BR}}^{X_1 X_2} &= \frac{\text{BR}(\phi \rightarrow X_1 X_2)}{\text{BR}(h \rightarrow X_1 X_2)}. \end{aligned} \quad (\text{C7})$$

Note that at the leading order of perturbative computations the ratios $r_{\text{contam}}(\phi)$ and $r_{\text{BR}}^{X_1 X_2}$ depend only on N_C once the Higgs mass is fixed to be 125 GeV. At the 8 TeV LHC, for $N_C = 4$ we have

$$\begin{aligned} r_{\text{contam}}(\phi) &\simeq 106, \\ r_{\text{BR}}^{WW} &= r_{\text{BR}}^{ZZ} = r_{\text{BR}}^{\tau\tau} \simeq 0.26, \quad r_{\text{BR}}^{\gamma\gamma} \simeq 0.37 \end{aligned} \quad (\text{C8})$$

and for the SM Higgs,

$$r_{\text{contam}}(h) \simeq 1.4. \quad (\text{C9})$$

Thus we estimate the signal strengths,

$$\begin{aligned} \mu_{VBF}^{WW/ZZ/\tau\tau} &\simeq 0.6 \left(\frac{v_{\text{EW}}/F_\phi}{0.23} \right)^2, \\ \mu_{VBF}^{\gamma\gamma} &\simeq 0.8 \left(\frac{v_{\text{EW}}/F_\phi}{0.23} \right)^2. \end{aligned} \quad (\text{C10})$$

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